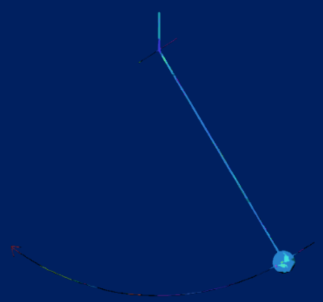
Revealing a Pre-Historic Measurement System

The measurement units used by pre-historic man to build stone circles and the pyramids have never been found because everyone who has ever looked for an ancient ruler has assumed that each ancient measurement system would have been based on a single measurement unit as it does today. Furthermore, many researchers seem to have had the idea that the ancient length they were looking for would be closely related to some known ancient measurement length and carried out their analyses by subjectively trying to make their preferred lengths fit the data. A more objective mathematical method has been adopted now which divides the measured radii of a large number of Scottish stone circles by integer multiples to give a series of lengths whose frequency of occurrence is analysed to reveal which of these lengths is likely to have been used as rulers to measure these circles. A series of twelve lengths emerges from the data that account for the dimensions of the different stone circles. These lengths form a measurement system where each measurement length is related to each other length through circular geometry so that one measurement length used as a radius of a circle results in a whole multiple of one of the other twelve lengths to describe the circumference of that circle. On closer examination it was found that these lengths used as pendulums, had periods of oscillation that gave whole thousands of swings for the time it took the Earth to rotate by integer multiples of Megalithic Degrees. The ancient measurement system revealed by the new analysis therefore appears to have integrated the measurement of both time and physical length in one ingenious measurement system. Some of the measurement lengths found were known ancient measurement lengths such as the Royal Cubit of 52.36cm, the Sacred Cubit of 63.66cm and the half a Yard of 45.75cm but contrary to our expectations, all were used simultaneously alongside the other nine measurement lengths. Remarkably one measurement length was precisely half a Metre or 50.0cm a length that is considered to be a modern length but used in Neolithic Scotland some 5000 years ago.



The Megalithic Yard - Revisited

Introduction

Many of the ancient stone circles of the British Isles and France were constructed some five thousand years ago by our Neolithic ancestors as true circles and ellipses. Given the size and consistent circular or elliptical shape of the rings it is likely that the plans for the stone circles were initially drawn out on the ground as circles scratched in the earth by rotating a string radius about a central point before erecting megaliths to stand on the perimeter of these marked circles. Many engineers and archaeologists have considered the possibility that these rings were measured on the ground using some unknown pre-historic measurement system to create rings of a desired size. The search to find the measurement units that may have been used in pre-historic times from the diameters of these stone circles has, to date, been of very limited success and as there are no artefacts that are recognisable as rulers that survive from this time there is no evidence for such a measurement system. The Scottish engineer and Oxford Professor Alexander Thom proposed that a unit of length he termed the Megalithic Yard had been used to measure the dimensions of stone circles as the analysis of surveys he personally made of hundreds of circles indicated a statistically significant fit for a ruler length of 2.72 feet or 83 centimetres. His proposal and statistical analysis have been questioned by many archaeologists and the attempts by other researchers to make their proposed measures fit the measured data of archaeological sites especially with regard to the dimensions of the stone circles and pyramids of Egypt has introduced an element of subjectivity into the science of metrology that has created an atmosphere amongst academics of great scepticism towards this area of investigation. However the fact remains that there are many hundreds of stone circle sites that survive in a reasonable state of preservation and if these monuments were laid out on the ground using a measuring system then there should be sufficient surviving dimensional evidence at these sites to determine both the likelihood that a prehistoric ruler was used and if so, what the length of that ruler was. The abundant stone circle dimensional data simply requires to be re-examined in a more thorough way.

Stone circles

The Scottish stone circles provide a very good data set for the re-examination of stone circle dimensions because they represent the highest concentration of stone circles in Northern Europe and there is a good chance that they were built by people using the same technology and were erected for the same purpose. The advantage of limiting ourselves to the examination of Scottish stone circles is also supported by the work of Kendall[1] and Freeman[2] who, when examining the data of Thom[3] regarding British stone circles and his suggestion of the existence of a prehistoric measuring unit he termed the "megalithic yard", concluded that there was statistical evidence of a uniform unit measure in Scottish circles but not in English circles.

Circle Making

The stone circles encompass a range of ring shapes that include circles, flattened circles, ellipses and egg-shaped rings. For the purpose of this study only stone circles that appear to be based on a true circle were examined as this shape results in a megalithic structure that can most easily and confidently be analysed in terms of its radius and circumference.

It is likely that a person wishing to construct a stone circle would first draw the circle on the ground using a length of string by scratching a circle in the earth. Perhaps the stone circle maker would firstly hammer a peg into an area of flat ground marking the centre of the circle and place a length of cord with a loop at its end over the peg and at the other end of the cord fix a stick to scratch a mark directly in the earth as the radial cord was first stretched taught and then rotated around the central peg to form a circle. Stones could then be sunk in the ground, spaced around the circle's perimeter at various positions to form the stone circles that we still see today some 5000 years after their construction. The question is whether the string used to draw out these circles was of a pre-determined measured length consisting of an integer multiple of some prehistoric measurement unit or whether the length of cord used was just an arbitrary length or a rough number of paces that varied from person to person and circle to circle. The stone placements may have given alignments with the rising and setting Sun on important festival days such as the winter and summer solstices but the precise purpose of the stone circles can be ignored for now as this present analysis is exclusively focussed on the dimensions of the stone circles. If the circle makers used a measuring system to construct their circles then integer multiples of the unit measurement lengths may have been used to determine the length of the cord used as the radius of the circle. It may also have been important to form a circle whose length of circumference was equivalent to an integer multiple of the measurement length. The ruinous state of many of the stone circles may introduce a level of uncertainty as to the intended dimensions of the original rings but there are very many circles that survive in a good state of preservation that allowed Professor Alexander Thom, a respected engineer and academic, to consider that the diameters of many of these monuments could be measured to an accuracy of within one foot and it is those monuments that were selected for analysis.

Unfortunately, the historical approach to trying to reveal a lost measurement unit has invariably involved selecting a known archaic unit length from another ancient civilisation and to try and match that measure to the dimensions of the archaeological structure under investigation. Part of the reason for this approach was that it was a commonly held view that the oldest developed civilisations were in Egypt, Arabia and Greece and that the technology must have spread outwards from these, civilised, technologically advanced hubs to peripheral cultures such as occurred in what was considered the "barbarian" North for example. The length of the chosen measure could then be adjusted to give a better-fit with the field data and then finally that proposed amended measure analysed statistically to test whether the fit was significant. In this way proposed archaic lengths such as the Pyramid inch and the Megalithic Yard came into being. This is a very narrow and limiting approach to investigation and misses the opportunity to reveal interesting and unexpected information that might present itself from a less restrictive analysis free from assumptions of the transfer of technology and the preconceived idea of what the measurement unit might be. Indeed, it is never good practise to set out to try and find something that you believe might be the answer because it is strange how the analysis and interpretation of results can bend to present you with exactly what you want to find. Far better to make the measurements and then objectively analyse that data to see if there are any significant measurement lengths that could explain the stone circle dimensions in terms of integer multiple units of those lengths and only then see if the lengths revealed by the mathematical analysis have any significance or relationship with other ancient known measures.

Thom's analysis of his stone circle dimensional data used the circle's diameter to describe the size of the circle. An alternative approach is to consider that since it is the radius that is important in physically drawing a circle rather than its diameter, perhaps it was the length of radius that was of prime importance in describing the circle's size, this may seem a moot point but it does have some significance in our analysis as an odd integer multiple length diameter would require a half fraction of the measurement unit to be used in forming the radius. Measurement data for a large sample of circles of known radii is required in order to determine whether certain multiple unit lengths or quanta can account for the stone circles measured dimensions and it is fortunate that Alexander Thom expended so much time and effort in accurately and consistently surveying so many stone circles using the same tried and tested manner. The preliminary process in analysing the data is to assume that only integer multiple units of a length were chosen to form the radius. Dividing each measured stone circle's radius by integers in the range 1-100 produces a table of lengths which may contain certain lengths that occur more frequently than by chance if a measuring system was used to construct these circles. A series of random circle sizes within the same size range as the stone circle sample can be generated for comparison together with an evaluation of standard deviation to determine the significance of any stone circle measurement lengths revealed by the process. The tables of results can be analysed by counting the number of times each length, generated by the division process, occurs. The process of counting considers any length within the range 300 - 2000mm at 1mm intervals as a potential measurement unit. The frequency of a length occurring as a result of dividing the radius by an integer is plotted against the length resulting from that division. If certain measurement lengths occur significantly more frequently than by chance, it is likely that a measurement unit was used to form these stone circles. This method seeks to determine two things, firstly whether there is any evidence for a common measurement unit used and secondly if there is, to determine what that measurement unit length was. The same process can be repeated on the circumferences of the stone circles to determine whether the length of circumference was likewise important to the architects of the stone circles.

Scottish Stone Circles

Alexander Thom professor of Engineering at Oxford University surveyed hundreds of stone circles throughout Britain and Northern France and his measurements provide us with the most extensive, accurate and consistent body of information regarding the dimensions and orientation of the stone circles. Thom appreciated the difficulties involved in trying to accurately determine the dimensions of stone circles situated in wild locations with tape measures, theodolites and prismatic compasses but his talent and experience as a surveyor provides us with a unique opportunity to obtain high quality dimensional data from a large number of stone circles obtained by one dedicated man and his family in a consistent and scientific manner. The clever method employed by Thom was to choose an arbitrary centre of the circle in the field and measure the distance from that point to the corners of the bases of each of the megaliths. The bearing of the corners was also determined so that accurate plans could later be drawn for the stone circle and the orientation of the circle relative to North accurately established. The actual original centre for each circle and diameter was later determined by firstly drawing the stone circle as measured in the field. As Thom describes his method " a carefully drawn circle is passed through the stones. The exact size chosen is unimportant as is also the position of the centre. Divide the ring into four quadrants. Mark what appears to be the centre of the base of each stone and measure the distance of this centre from the circle: positive if the stone centre is outside the circle, negative when it is inside. Find the mean for each quadrant separately. Call these means δ_{ne} , δ_{se} , δ_{sw} and δ_{nw}. Then the required diameter is the diameter of the superimposed circle increased by

$$1/2(\delta_{ne} + \delta_{se} + \delta_{sw} + \delta_{nw})$$

The chosen centre of the superimposed should now be moved to the north-east by

$1/2(\delta_{ne} - \delta_{sw})$ and to the north-west by $1/2(\delta_{nw} - \delta_{se})$. [5] "

The diameter of each circle was therefore reported by Thom as the distance from the centre or midpoint of the stones standing directly opposite each other on the perimeter of the circle as opposed to either an internal diameter between the inner stone faces or an external diameter between the outer stone faces. In otherwords, it has been assumed that each megalith has been sunk into the ground so that its base was bisected by the circle's circumference as opposed to planting megaliths so that either their inner face or outer face aligned with the circumference drawn on the ground. The importance of this assumption cannot be overstated for the thickness of the megaliths is significant and if incorrect, its inclusion overestimates the intended diameter by over one foot. Thom's argument for a circumference that involves the centres of the stones is largely due to his admitted presumption from the outset that the megalithic yard had been used to construct these monuments and considering the diameter between the mid-point of the stones, resulted in a length that gave a closer match to a multiple of the proposed Megalithic Yard measure that was being searched for than data based on the diameter as measured between the internal or external faces of the stones. Whilst megaliths may have fallen and moved over the millennia, in general the pattern observed for stones that have been erected very close to each other is one where their inner faces are orientated to align with each other, furthermore with stones of different thicknesses the general appearance appears to be one of an alignment that creates a smooth internal curve of their inner faces which would imply that these stones have not been erected to sit on a ring that bisected the stone but rather for the stone to be sunk into the ground by digging a hole outside the circle that reached the line of the circumference and allowed the inner face of the planted stone to touch the circle's circumference without crossing it.

Thom was convinced that the stone circles were constructed using a measurement length of 82.9cm which he dubbed "the Megalithic Yard". His statistical analysis using the lumped variance test of J.R. Broadbent[4] on his megalithic yard quantum suggested that the results of the analysis was significant.

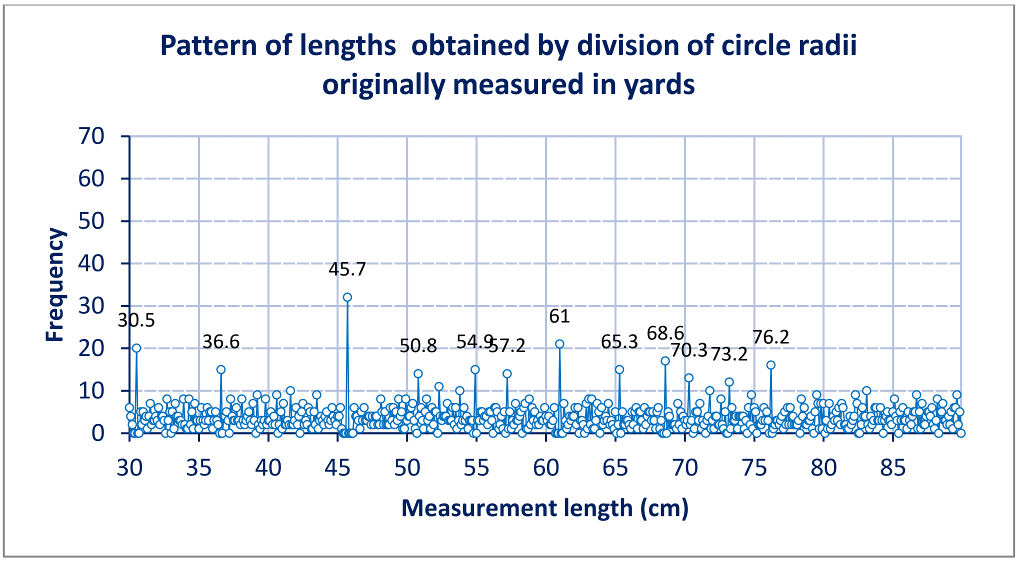
Others have challenged his statistical analysis and suggested that Thom's data could just as easily be explained as the average length of a pace. The jury remains undecided but perhaps a slightly different approach to the processing of Thom's excellent survey data will result in a more conclusive result.

Analysing a group of circles of known dimensions

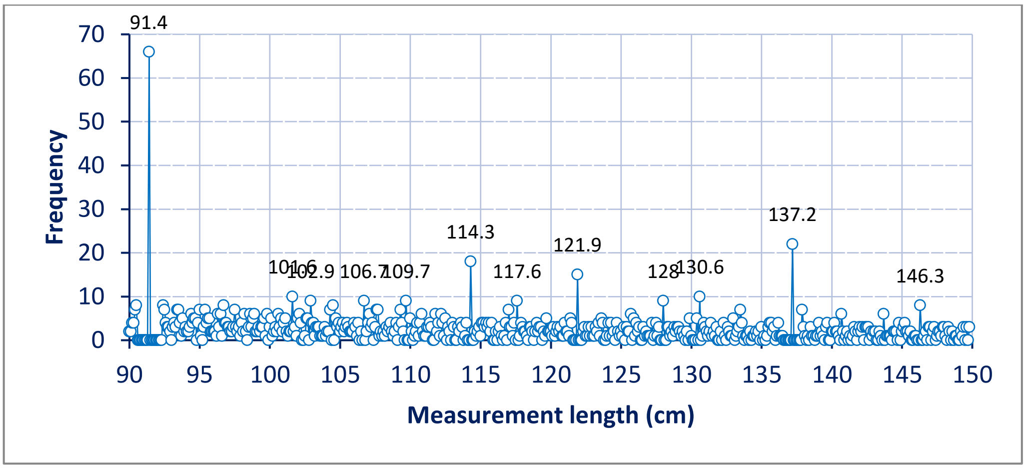
The challenge of trying to reveal an archaic measurement length from measurements described in modern units of length can be replicated by generating a random group of imaginary circles of different sizes but all formed using a radius consisting of integer multiples of a previously used but now redundant unit length. In this case the yard (36 inches) was chosen as the unit measure and then a group of randomly sized circles was generated based on radii of multiples of this unit before converting the radius of each of the imaginary circles into the metric system. In this way we know what we are trying to find but the processing of the data will show us the patterns we might expect to observe in the case where an unknown measurement unit has been used as in the case of the megalithic stone circle analysis.

Method for Revealing the unknown Base Measurement Unit used to measure the Radius of a Circle

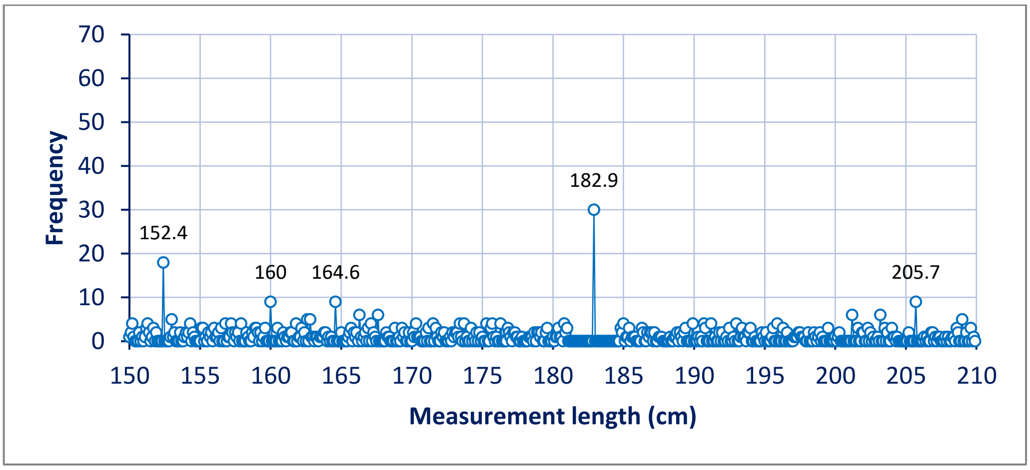
The yard (3 feet or 36 inches), a measurement unit that is no longer in common usage in Britain was used to construct a series of imaginary circles whose radii consisted of an integer multiple of yards. The radii of sixty such circles with randomly generated radii within the range 4-98 yards were converted to radial measurements expressed as centimetres to imitate our situation of trying to rediscover an unknown archaic measurement length from dimensional data for the circles obtained using the currently used metric measurement system. The converted radii measures were each divided by integers in the range 1-100 to obtain a series of one hundred lengths for each of the sixty circles. A table constructed of the six thousand lengths resulting from the process of division, can then be analysed to determine the most frequently occurring lengths. The frequency of occurrence of each length when plotted results in a curve that has a multitude of peaks. The largest peak representing the most frequently occurring length is equivalent to the measurement unit originally used to construct the circle, in this case 91.4cm (equivalent to one Yard) but there are also many other significant peaks representing half a yard and two yards, and thirds, quarters, fifths of yards as fractional and multiple values of this base length that occur as significant, but diminishing peaks. In the case where a measurement unit has been used to construct a group of stone circles, we would expect to see a similar series of peaks that are related to the base measurement used to measure those circles. The processing of the field data obtained from the actual stone circles will be complicated by the limitations in the accuracy of the measurements obtained unlike this example where exact integer multiples of the yard formed the data base for analysis.



Graph 1 (i)



Graph 1 (ii)



Graph 1(iii) Graphs 1(i)- 1(iii)show the frequency of occurrence of lengths resulting from division of randomly sized circles, whose radii were measured using yards, by integers 1-100.

Peak (cm)	Fraction of a Yard
30.5	1/3
36.6	2/5
45.7	1/2
50.8	11/20
54.9	3/5
57.2	5/8
61.0	2/3
65.3	5/7
68.6	3/4
70.3	10/13
73.2	4/5
76.2	5/6
91.4	**1/1**
101.6	11/10
102.9	9/8
106.7	7/6
109.7	6/5
114.3	5/4
117.6	9/7
121.9	4/3
128.0	7/5
130.6	10/7
137.2	3/2
146.3	8/5
152.4	5/3
160.0	7/4
164.6	9/5
182.9	2/1
205.7	9/4

Table 1 shows how the peaks observed in Graph 1 (i-iii) relate to fractions of a yard

Measurement of Scottish Stone Circles

The same process of division of the dimensions of Scottish stone circles can be carried out using the field measurements obtained by Thom. In this way if some measurement unit was used to determine the radius of the stone circles then we might expect to see a similar pattern of peaks associated with that measurement length. Sixty-six stone circles surveyed by Thom whose diameters he considered to be accurate to within a foot were selected for analysis. The plans for these stone circles and many others from England and Wales are detailed in the British Archaeological Reports (BAR), British Series 81, 1980 Megalithic Rings by A and A.S. Thom collated by A. Burl. However rather than using the diameters of these 66 stone circles reported by Thom based on the distance between the mid points of the megaliths, the stone circle internal radii were determined manually using digital callipers directly from Thom's carefully drawn plans using the scale drawn on each plan to convert the calliper reading to a length expressed in centimetres. The sixty-six stone circles and their measured internal radii are listed below together for comparison with the diameters determined by Thom which he calculated to the centre of the stones' base.

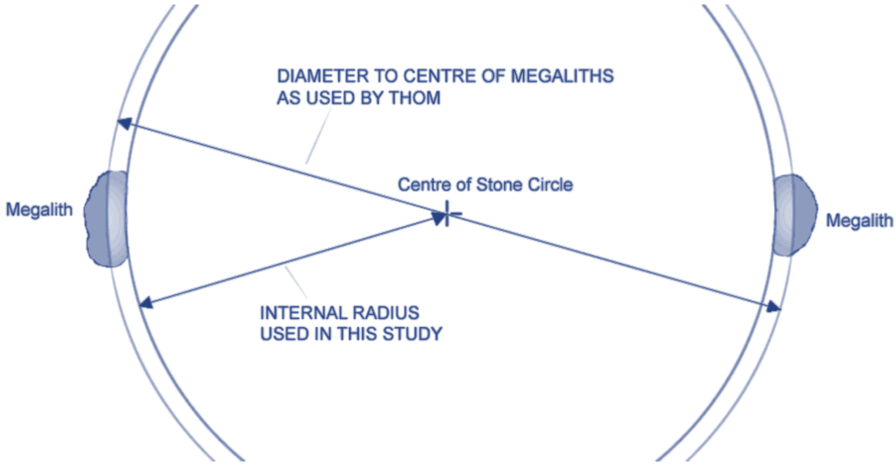


Diagram 1 illustrates the different dimensions used for analysis by Thom and the author

The information and measurements described in Table 2 were taken from A. and A.S. Thom, Megalithic Rings collated by A. Burl, British Archaeological Report 81, 1980. The internal radii were measured by the author from the plans using a digital calliper. The diameters quoted are those estimated by Thom in feet for the diameter relating to the circumference drawn by Thom that bisected the base of the megaliths.

Stone Circle	Site Ref (Thom)	Map Reference	Internal Radius Measured from Plans (cm)	Diameter (stone centres Thom) (ft)	Equivalent Radius (to stone centre) Thom (cm)
Shin River	N 2/3	NC 582049	191.0	13.6	207.3
Monzie	P 1/13	NN 882242	216.3	16.4	249.9
Fountain Hill	B 1/10	NJ 880328	252.2	16.9	257.6
Shin River	N 2/3	NC 582049	291.2	20.5	312.4
Loch Mannoch	G 4/9	NX 661614	301.3	21.0	339.6
Esslie Greater	B 2/4	NO 717916	314.0	20.6 (Burl)	313.9
Burreldales	B 4/2	NJ 676550	315.1	21.8	327.6
Miltown of Clava	B 7/2	NH 751438	318.5	22.0	335.3
The Mound	N 2/2	NH 770991	328.9	24.5	373.4
Shianbank	P 2/8	NO 156272	380.5	27.5	419.1
Shianbank	P 2/8	NO 156272	382.3	27.5	419.1
Carnousie	B 4/1	NJ 678505	392.2	27.0	411.5
Blindwells	P 2/3	NO 125314	394.7	28.0	426.7
Colen	P 2/6	NO 110311	396.0	28.0	426.7
River Ness	B 7/19	NH 621388	433.0	30.1	458.7
Cullerlie	B 2/7	NJ 785043	486.0	33.3	507.5
Raedykes North	B 3/4	NO 832907	487.2	32.6	496.8
Farr Post Office	B 7/17	NH 682332	487.7	32.0	487.7
Little Urchany	B 6/1	NH 866486	526.0	35.4	538.9
Ardlair	B 1/18	NJ 553280	534.1	37.6	573.0
Wamphray	G 7/3	NY 140960	539.7	37.0	563.9
Dalcross Castle	B 7/6	NH 780484	576.1	39.2	597.4
Aviemore	B 7/12	NH 896134	636.6	43.0	655.3
Loch Buie	M 2/14	NM 618251	641.0	44.1	672.1
Esslie the Lesser	B 2/5	NO 722921	648.0	43.5	662.9
Auchnagorth	B 1/5	NJ 839563	650.7	44.9	684.3
Tomnagorn	B 2/16	NJ 651077	674.7	46.8	712.5
Templewood	A 2/8	NR 827979	676.2	44.5	678.2
Leys of Marlee	P 2/1	NO 160439	699.9	49.4	752.9
Westerton	B 1/16	NJ 706190	734.0	49.5	754.4
Aquorthies Kingausie	B 3/1	NO 902963	741.6	49.7	757.4
Sheldon of Bourtie	B 1/8	NJ 823249	781.3	53.0	807.7
Loanhead of Daviot	B 1/26	NJ 748289	809.0	54.4	829.1
Clune Wood	B 3/7	NO 795950	837.7	56.4	859.5
Midmar	B 2/17	NJ 699064	837.8	56.8	865.6
Tomnaverie	B 2/9	NJ 487034	838.4	56.0*	853.4
Miltown of Clava	B 7/2	NH 751438	850.8	59.1	900.7
Yonder Bognie	B 1/23	NJ 600458	864.3	57.0	883.9
Esslie the Greater	B 2/4	NO 717916	880.0	59.8	912.0

Stone Circle	Site Ref (Thom)	Map Reference	Internal Radius Measured from Plans (cm)	Diameter (stone centres Thom) (ft)	Equivalent Radius (to stone centre) Thom (cm)
Tyrebagger	B 2/1	NJ 859132	**888.0**	59.3	903.7
Easter Aquorthies	B 1/6	NJ 733207	**928.0**	64.0	960.1
Moyness	B 6/2	NH 951536	**932.0**	62.6	953.4
Loch Nell	A 1/2	NM 906291	**972.9**	65.1	990.6
Castle Fraser	B 2/3	NJ 715124	**973.3**	66.9	1018.0
West Farr	B 7/16	NH 680335	**989.6**	66.8	1018.0
Loanhead of Daviot	B 1/26	NJ 748289	**1001.4**	68.0	1036.3
Little Urchany	B 6/1	NH 866482	**1017.8**	68.0	1036.3
River Ness	B 7/19	NH 621380	**1020.0**	69.1	1053.1
Tomnagorn	B2/16	NJ 651077	**1075.0**	73.3	1117.5
Tarland	B 2/8	NJ 471052	**1100.0**	74.1	1129.3
Aquorthies Kingausie	B 3/1	NO 902963	**1115.1**	75.1	1144.5
Druid Temple	B 7/18	NH 685420	**1129.3**	74.3	1132.3
Aviemore	B 7/12	NH 896134	**1131.0**	76.0	1158.2
Esslie the Greater	B 2/4	NO 717916	**1152.0**	76.2	1160.7
Mains of Gask	B 7/15	NH 680359	**1223.0**	82.9	1263.4
Sunhoney	B 2/2	NJ 716057	**1241.8**	83.3	1264.9
Carnousie	B 4/1	NJ 678505	**1242.9**	84.0	1280.2
Drannandow	G 4/3	NX 400710	**1338.2**	89.1	1357.9
Millton	B 4/4	NJ 550487	**1344.0**	92.0	1397.5
Clava South	B 7/1	NH 757444	**1547.8**	103.9	1586.5
Clava Middle	B 7/1	NH 757444	**1548.5**	104.2	1597.2
Urquhart	B 5/1	NJ 290640	**1616.6**	110.0	1676.4
Sheldon of Bourtie	B 1/8	NJ 823249	**1677.7**	108.4	1652.0
West Farr	B 7/16	NH 680335	**1689.7**	113.2	1725.2
Mains of Gask	B 7/15	NH 680359	**1777.0**	119.9	1827.3
Latheron Wheel	N 1/13	ND 180350	**2827.0**	188.3	2869.7

Table 2. The names, dimensions and Grid references of the Scottish stone circles used in this study.

There is a mean difference of about 27cm between the internal radius measurement obtained from the plans of the stone circles drawn in the British Archaeological Report 81 and half the value given by Thom for the diameters. This difference results mainly from the fact that Thom measured the diameter as the distance between the centre of the stones and we have considered the internal radius or the distance from the centre of the circle to the inner face of the standing stones. The difference of 27cm therefore corresponds to approximately half the mean thickness (54cm) of the megaliths used to construct the rings.

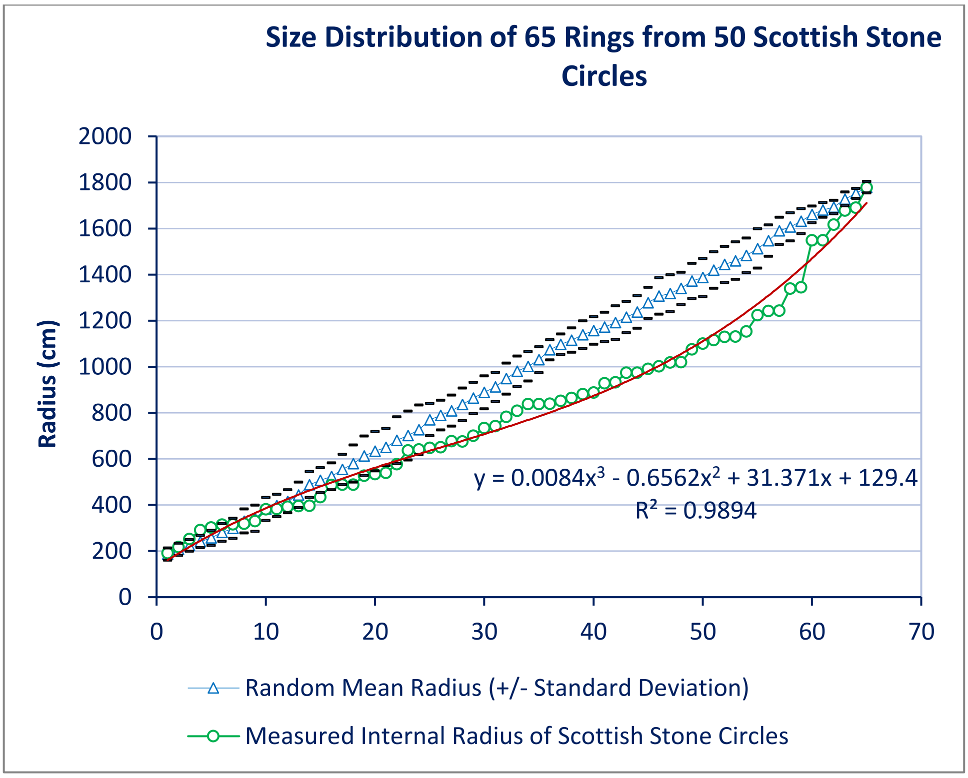
Locations of the Stone Circles

Diagram 2 Stone circles used in this analysis. Reference numbers are those given by Thom.

There are 124 Scottish stone circle plans drawn by Thom in BAR 81, but of these only 51 are considered by Thom as being in a state that allow their diameters to be determined to an accuracy of within one foot. The locations of those 51 stone circles comprising 66 circular rings, because some stone circles have more than one ring, are shown in Diagram 2. The majority of the stone circles are located in the North East of Scotland in Aberdeenshire, Kincardineshire, Banffshire, Morayshire, Inverness-shire and Perthshire.

Size distribution of Stone Circles

Examining the pattern of internal radii of the stone circles reveals that whilst randomly generated circle radii within the same size range as the stone circles produce a curve that tends towards a straight line, the dimensions of the internal radii of the stone circles deviates significantly from the random line as it forms a curve that lies well beneath it lying outside the range of standard deviation of the random circle curve indicating that the size of the circles is generally smaller than would be expected if the choice of circle size was a random process. Moreover, the pattern of the curve lying beneath the random mean curve is step-like and can be accounted for by the individual points on the measured internal radius curve forming a series of horizontal steps where certain radii occur multiple times. The fact that some stone circles in geographically remote locations separated by many miles had identical sizes suggests that certain radii were preferred and that some measurement system was likely to have been used.



Graph 2

The fifty stone circles are those detailed in Table 1 with the exception of Latheron Wheel which was omitted due to its atypical much larger radius. The trend-line drawn for the stone circle curve is a third order polynomial. The random circle curve is formed from ten sets of random circles generated within the same size range as the 50 selected stone circles.

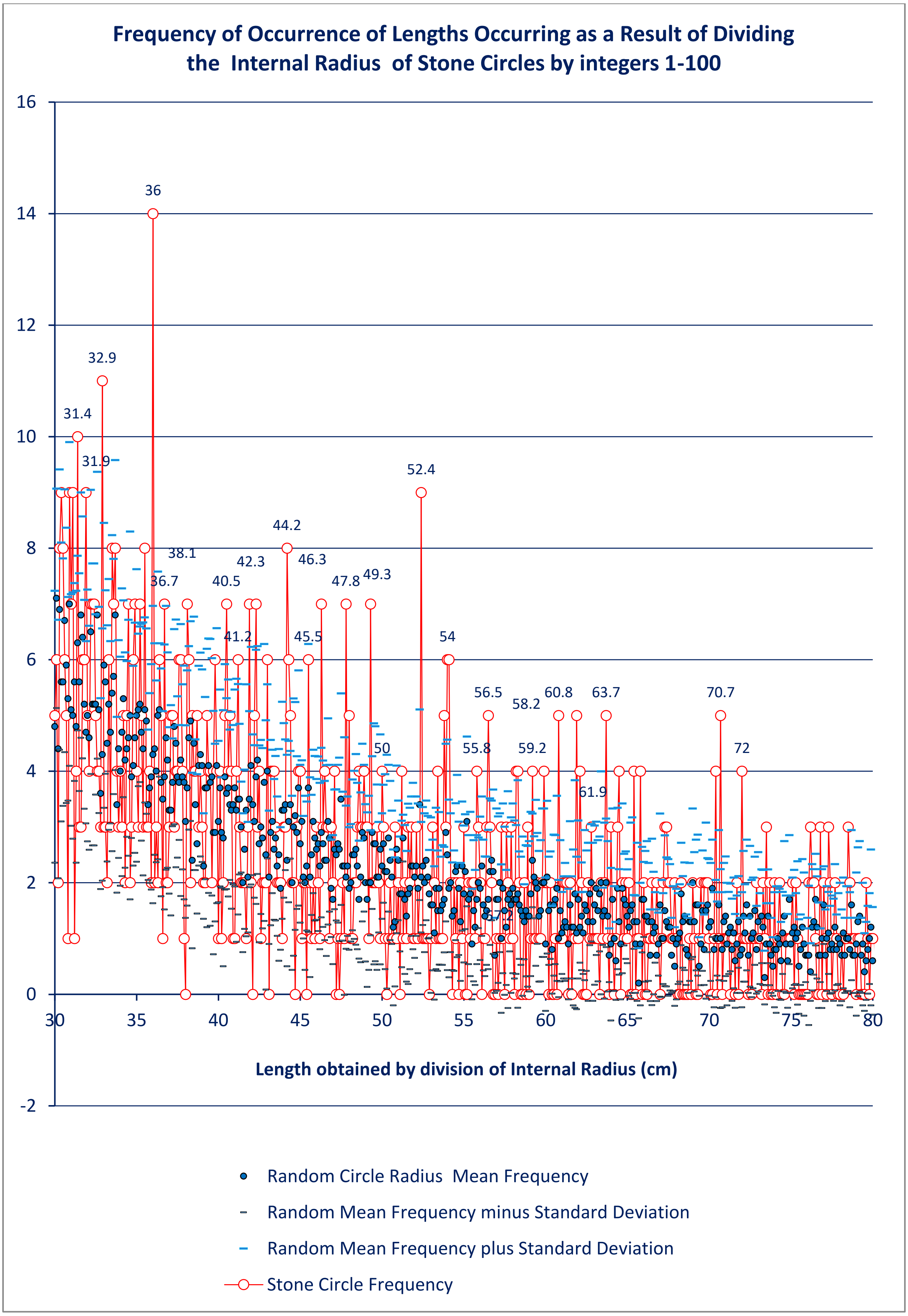
Random Circles

In order to determine the significance of the dimensions of the selected 66 stone circle rings a set of random circles was formed for comparison. The first set of random circles was generated using random values of internal radii within the range 191cm - 1777cm corresponding to the approximate range of dimensions determined for the actual stone circles, with the exception of the Latheron Wheel stone circle which being much larger than the next largest circle was omitted to prevent skewing the curve.

Measurement Units Used to Plan the Scottish Stone Circles

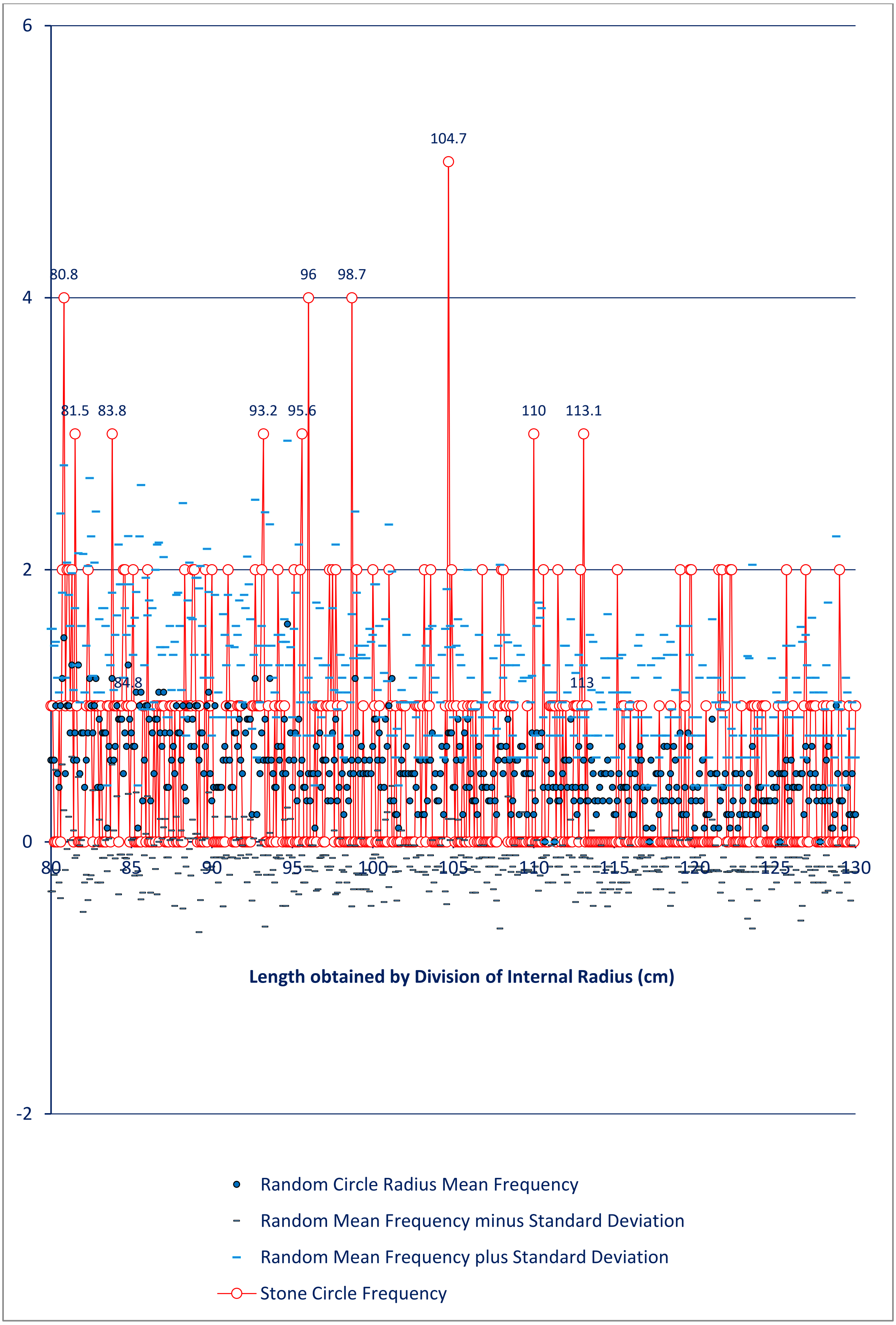
The internal radii measurements determined from the plans of the 66 rings detailed in Table 1 were subjected to the same process of division by whole numbers as was applied to the example of the imaginary group of circles whose radii consisted of multiple lengths of yards. Each stone ring radius was divided in turn by all the integers in the range 1-100 and the lengths resulting from those divisions tabulated. The frequency of occurrence of each of the resulting lengths in the range 30cm to 200cm was counted for all 66 rings and a graph plotted. A new random set of circles was created to determine the significance of the pattern of peaks revealed. In order to achieve a random set of circles that conformed to the same general pattern of size distribution as the stone circles, the new set of random circles were generated using the trend-line polynomial equation that approximated the size distribution curve of the stone circles as shown in Graph 2, though not its step-like nature.

$$y = 0.0084x^3 - 0.6562x^2 + 31.371x + 129.41$$

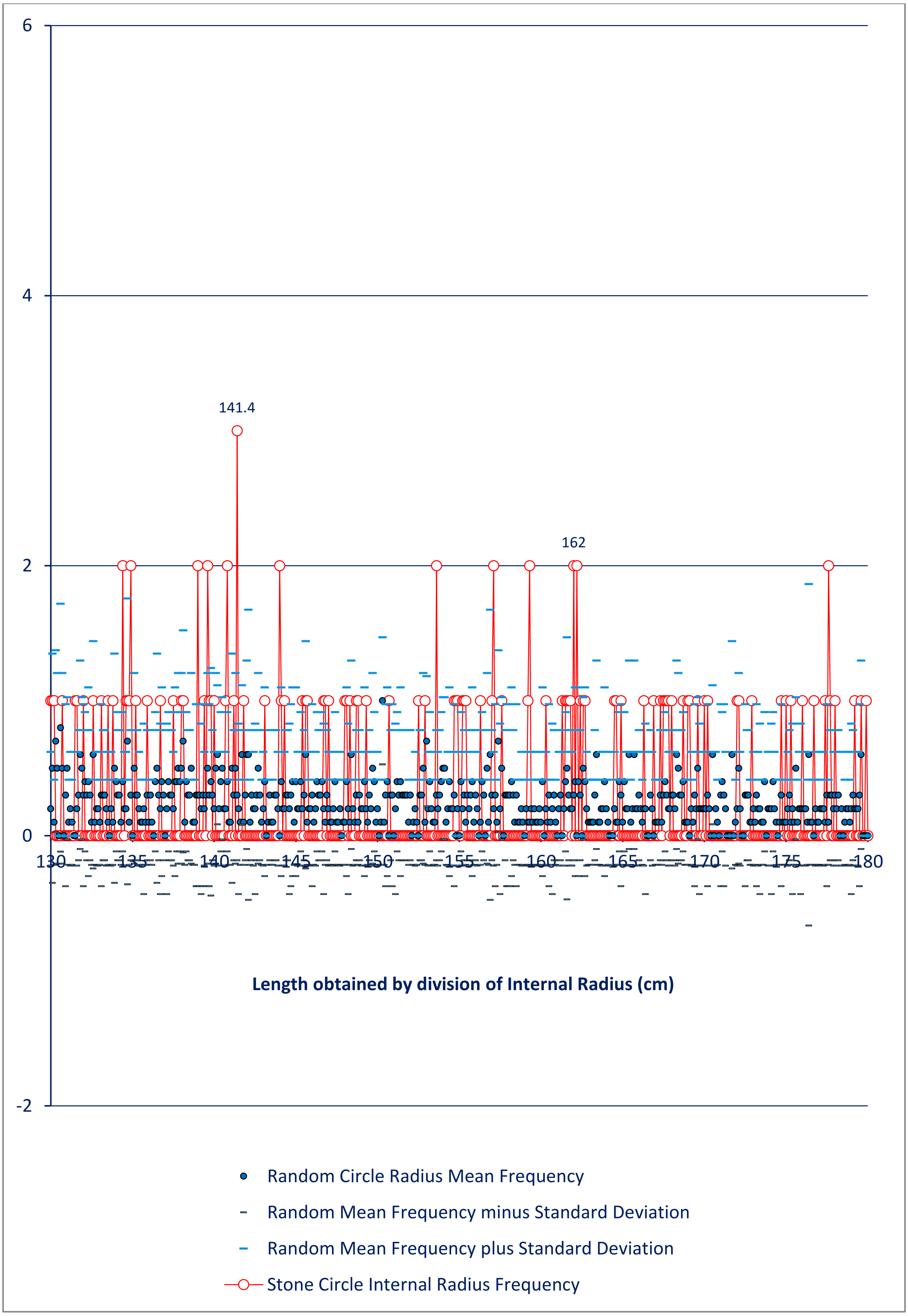


Frequency of Occurrence of Lengths Occurring as a Result of Dividing the Internal Radius of Stone Circles by integers 1-100
16
14
12
10
8
6
4
2
0
-2
30
35
40
45
50
55
60
65
70
75
80
36
32.9
31.4
31.9
52.4
44.2
38.1
42.3
46.3
36.7
40.5
47.8
49.3
41.2
45.5
54
56.5
58.2
60.8
63.7
70.7
50
55.8
59.2
72
61.9
Length obtained by division of Internal Radius (cm)
Random Circle Radius Mean Frequency
Random Mean Frequency minus Standard Deviation
Random Mean Frequency plus Standard Deviation
Stone Circle Frequency

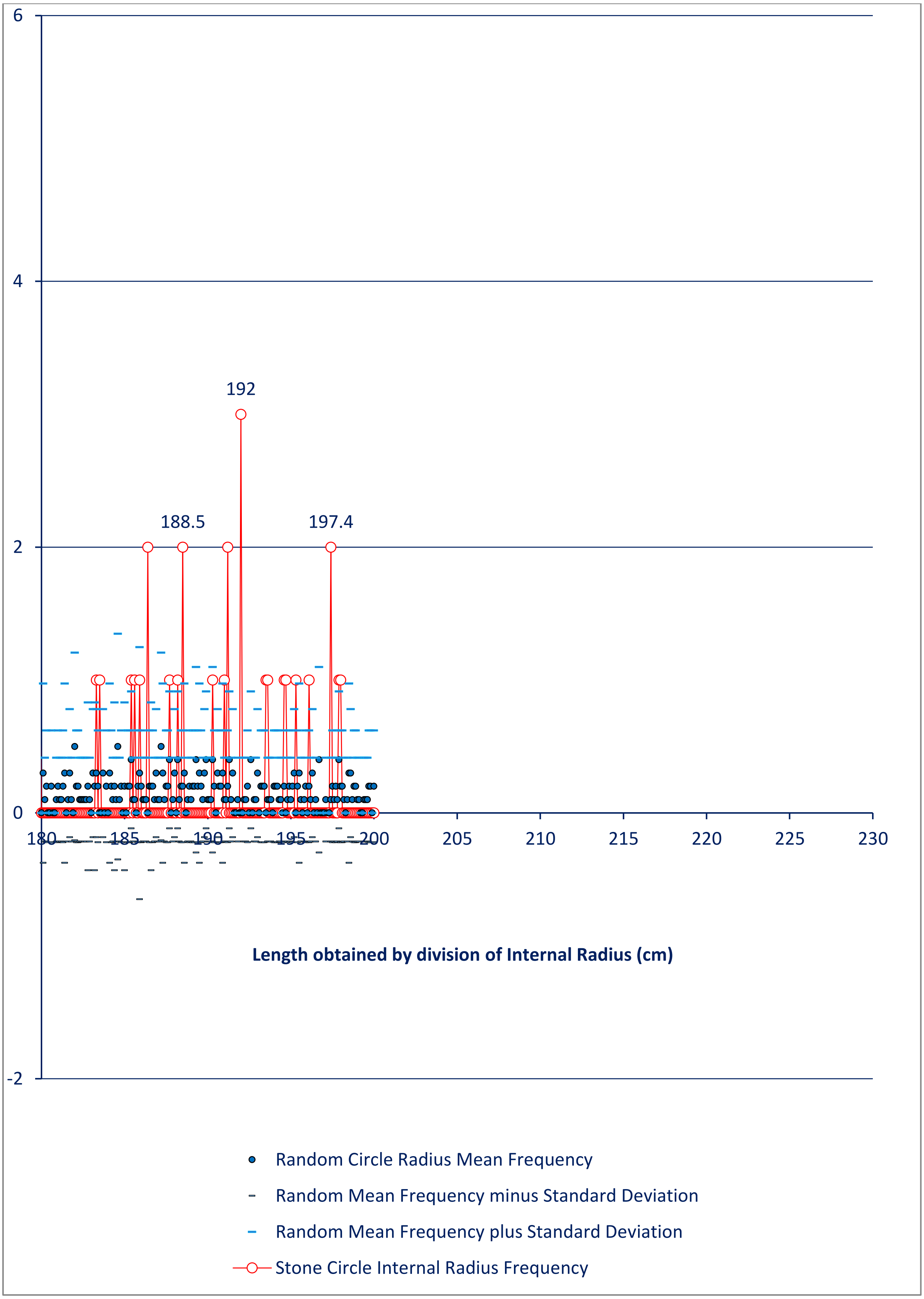
Graph 3 (i)


6
4
2
0
-2
104.7
80.8
96
98.7
81.5
83.8
93.2
95.6
110
113.1
84.8
113
80
85
90
95
100
105
110
115
120
125
130
Length obtained by Division of Internal Radius (cm)
Random Circle Radius Mean Frequency
Random Mean Frequency minus Standard Deviation
Random Mean Frequency plus Standard Deviation
Stone Circle Frequency

Graph 3 (ii)


6
4
2
0
-2
141.4
162
130
135
140
145
150
155
160
165
170
175
180
Length obtained by division of Internal Radius (cm)
Random Circle Radius Mean Frequency
Random Mean Frequency minus Standard Deviation
Random Mean Frequency plus Standard Deviation
Stone Circle Internal Radius Frequency

Graph 3 (iii)



6
4
2
0
-2
192
188.5
197.4
180
185
190
195
200
205
210
215
220
225
230
Length obtained by division of Internal Radius (cm)
Random Circle Radius Mean Frequency
Random Mean Frequency minus Standard Deviation
Random Mean Frequency plus Standard Deviation
Stone Circle Internal Radius Frequency

Graph 3 (iv)

Graphs 3 (i), 3(ii), 3(iii) and 3(iv) reveal a complicated pattern of multiple peaks of varying amplitude. There are many peaks from the stone circle analysis which lie outside the range of standard deviation for the randomly generated circles. These peaks represent lengths which occur significantly more frequently than the randomly generated circle data resulting in a graph that shares the same general pattern seen in Graph 1 suggesting that some measurement length was used to measure the stone circle radii. The graphs differ from Graph 1 in that there are many more peaks present in Graph 3 than might be expected if only one measurement unit had been used suggesting that more than one unit length was employed by the stone circle builders. The pattern of peaks observed in Graph 1 was easily interpreted as a major peak equivalent to the yard base measurement used to construct the series of theoretical circles accompanied by lesser peaks representing fractional and multiple values of the yard. The significance of the different peaks revealed in Graph 3 can be assessed by comparing the frequency of occurrence of the measurement lengths associated with each peak with the mean frequency value obtained for the randomly generated circles within the same size range as the stone circles. There are many peaks (283 or 16.7% of the data) that lie above the random mean frequency plus standard deviation value suggesting that these lengths are significant, if more than one unit measure was used then a series of associated peaks representing fractional and multiple values of the base unit lengths may account for the large number of peaks observed. To untangle the data and process the large number of peaks to reveal any hidden base unit lengths, the number of peaks examined can be reduced by over 80% by considering only those lengths that lie above the random circle mean plus standard deviation, that occur four or more times, and whose frequency of occurrence is greater than or equal to 1.7 times the frequency of occurrence of the random circle mean for the same length. Table 2 shows the forty-six lengths that satisfy these conditions.

Peak Length (cm) Obtained by Division of Internal Radius by integer values	Frequency of Occurrence of Length (F)	(F) /Random Mean Frequency (F_{rm}) + Standard Deviation	(F) / Random Mean Frequency (F_{rm})
31.1	9	1.2	1.8
31.9	9	1.4	1.9
32.9	11	1.7	2.6
36.0	14	2.0	3.3
36.7	7	1.1	1.8
38.1	7	1.2	1.9
38.4	5	1.3	2.1
38.7	5	1.3	1.9
39.8	6	1.3	2.1
40.5	7	1.3	1.9
41.2	6	1.1	1.7
41.9	7	1.4	2.0
42.3	7	1.1	1.8
44.2	8	1.9	3.3
44.3	6	1.3	1.9
46.3	7	1.6	2.4
47.8	7	2.1	3.3
48.0	5	1.7	2.6
48.6	4	1.5	2.4
49.3	7	2.4	3.5
50.0	4	1.2	1.7
51.2	4	1.3	2.2
52.4	9	2.9	4.3
53.4	4	1.6	2.7
53.8	5	1.9	2.9
54.0	6	1.3	2.4
54.1	6	1.7	3.2
55.8	4	1.4	2.4
56.5	5	1.7	2.9
58.2	4	1.3	2.4
58.3	4	1.0	2.2

Table 2 (i)

Peak Length (cm) Obtained by Division of Internal Radius by integer values	Frequency of Occurrence of Length (F)	(F) /Random Mean Frequency (F_{rm}) + Standard Deviation	(F) / Random Mean Frequency (F_{rm})
59.9	4	1.2	2.0
60.8	5	2.4	5.0
61.9	5	2.0	4.5
62.1	4	2.2	3.3
63.7	5	1.6	2.5
64.5	4	1.8	3.1
65.4	4	2.2	3.1
65.8	4	2.0	4.4
70.4	4	1.9	4.0
70.7	5	2.2	5.0
72.0	4	2.2	5.0
80.8	4	1.4	2.7
96.0	4	3.3	8.0
98.7	4	2.4	6.7
104.7	5	3.5	6.3

Table 2 (ii) lengths corresponding to peaks from Graph 3 that occur outside the range of standard deviation for the random circle lengths, which occur four or more times, and whose frequency of occurrence is greater than or equal to 1.7 times the frequency of occurrence of the random circle mean.

There appear to be relationships between the various peak lengths for example 36.0, 54.0, 72.0 are related through the ratios 1 : 1.5 : 2.0 and 52.4, 104.7 through 1 : 2. This is consistent with the pattern of peaks seen in Graph 1 with the yard data. The relationships can be further explored by dividing each length by all the other lengths to reveal the base lengths and their fractional length.

Peak Length (cm) (F>3, F/F_{rm}>1.7)	Relationship with Base Units	Relationship with other Peak Lengths
31.1	41.2x 3/4, 46.3x 2/3	62.1 x 1/2
31.9	36.0x 8/9, 41.2x7/9, 63.7x1/2, 80.8x 2/5	39.8x 4/5,42.3x 3/4, 47.8x 2/3, 53.4x 3/5, 96x 1/3
32.9	41.2 x 4/5	49.3x 2/3, 65.8x 1/2, 98.7x 1/3
36.0	36.0x 1, 58.2x Phi	48.0x 3/4, 54.0x 2/3, 59.9x3/5, 72.0x 1/2
36.7	52.4x 7/10	60.8x 3/5
38.1	63.7x 3/5	42.3x 9/10, 47.8x 4/5, 96.0x 2/5
38.4	58.2x 2/3, 63.7x 3/5	48.0x 4/5, 51.2x 3/4, 64.5x 3/5, 96.0x 2/5
38.7	58.2x 2/3	48.6x 4/5, 51.2x3/4, 62.1x Phi, 96.0x 2/5
39.8	50.0x 4/5	31.9x 5/4, 59.9x 2/3, 65.8x 3/5, 98.7x 2/5
40.5	36.0x 9/8, 80.8x 1/2	48.6x 5/6, 53.8x 3/4, 60.8x 2/3
41.2	41.2x 1, 46.3x 8/9	61.9x 2/3
41.9	52.4x 4/5	55.8x 3/4, 104.7x 2/5
42.3	56.5x 3/4, 63.7x 2/3	70.4x 3/5
44.2	50.0x 8/9	
44.3	50.0x 8/9	
46.3	46.3x 1, 52.4x 8/9, 58.2x 4/5	38.7x 6/5, 61.9x 3/4
47.8	63.7x 3/4	31.9x 3/2, 38.1x 5/4, 39.8x6/5, 59.9x 4/5, 72x 2/3, 96.0x 1/2
48.0	36.0x 4/3, 63.7x 3/4	31.9x 3/2, 38.4x 5/4, 59.9x 4/5, 72.0x 2/3, 96.0x 1/2
48.6	58.3x 5/6, 80.8x 3/5	38.7x 5/4, 40.5x 6/5, 60.8x 4/5, 64.5x 3/4
49.3	41.2x 6/5	32.9x 3/2, 61.9x 4/5, 65.8x3/4, 98.7x 1/2
50.0	50.0x 1, 80.8x Phi	
51.2	63.7x 4/5	38.4x 4/3
52.4	52.4x 1, 58.2x 9/10	41.9x 5/4, 65.4x 4/5, 104.7x 1/2
53.4	63.7x 5/6	38.1x 7/5, 70.7x 3/4
53.8	36.0x 3/2, 80.8x 2/3	38.4x 7/5
54.0	36.0x 3/2, 80.8x 2/3	40.5x 4/3, 72.0x 3/4, 38.7x 7/5
54.1	36.0x 3/2, 80.8x 2/3	40.5x 4/3, 72.0x 3/4, 38.7x 7/5

Table 3 (i)

Peak Length (cm) (F>3, F/F_{rm}>1.7)	Relationship with Base Units	Relationship with other Peak Lengths
55.8	46.3x 6/5	41.9x 4/3
56.5	56.5x 1, 63.7x 8/9	42.3x 4/3, 70.7x 4/5
58.2	36.0 x 1/Phi, 58.2x 1	38.7x3/2, 65.4x 8/9
58.3	36.0 x 1/Phi, 58.2x 1	48.6x 6/5
59.9	36.0x 5/3, 50.0x 6/5	39.8x 3/2, 47.8x 5/4
60.8	80.8x 3/4	40.5x 3/2, 48.6x 5/4
61.9	41.2x 3/2, 46.3x 4/3	38.7x 8/5, 44.2x 7/5, 49.3x 5/4
62.1	41.2x 3/2, 46.3x 4/3, 50.0x 5/4	
63.7	63.7x 1	31.9x 2, 39.8x 8/5, 42.3x 3/2, 47.8x 4/3, 96.0x 2/3
64.5	36.0x 9/5, 46.3x 7/5, 80.8x 4/5	38.7x 5/3, 48.6x 4/3
65.4	52.4x 5/4	98.7x 2/3
65.8	41.2x 8/5	32.9x 2, 49.3x 4/3, 98.7x 2/3
70.4	56.5x 5/4	38.4x 11/6, 42.3x 5/3
70.7	56.5x 5/4	44.2x 8/5
72.0	36.0x 2, 41.2x 7/4	48.0x 3/2, 54.0x 4/3, 59.9x 6/5, 96.0x 3/4
80.8	46.3x 7/4, 50.0x 1/Phi, 80.8x 1	31.1x 13/5, 36.7x 11/5, 40.5x 2, 48.6x 5/3, 53.8x 3/2, 64.5x 5/4
96.0	36.0x 8/3, 41.2x 7/3, 63.7x 3/2, 63.7x 3/2	31.9x 3, 38.4x 5/2, 48.0x 2, 53.4x 9/5, 72.0x 4/3
98.7	41.2x 12/5, 56.5x 7/4	32.9x 3, 42.3x 7/3, 49.3x 2, 65.8x 3/2
104.7	52.4x 2, 58.2x 9/5	41.9x 5/2, 59.9x 7/4, 65.4x 8/5

Table 3 (ii)

Each length can be described as a simple fractional multiple of one of the other lengths. There are though not one but nine base lengths; 36.0, 41.2, 46.3, 50.0, 52.4, 56.5, 58.2, 63.7 and 80.8cm that can be used to describe all the peaks revealed from the analysis and these may represent the measurement lengths used to construct the stone circles.

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
36.0	31.9	8/9
	36.0	1
	40.5	9/8
	48.0	4/3
	53.8	3/2
	54.0	3/2
	58.2	1/Phi
	59.9	5/3
	64.5	9/5
	72.0	2
	96.0	8/3

Table 4(i)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
41.2	31.1	3/4
	32.9	4/5
	41.2	1
	49.3	6/5
	61.9	3/2
	62.1	3/2
	65.8	8/5
	96.0	7/3
	98.7	12/5

Table 4 (ii)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
46.3	31.1	2/3
	41.2	8/9
	46.3	1
	55.8	6/5
	61.9	4/3
	62.1	4/3
	64.5	7/5
	80.8	7/4

Table 4 (iii)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
50.0	39.8	4/5
	44.2	8/9
	44.3	8/9
	50.0	1
	59.9	6/5
	80.8	1/Phi

Table 4 (iv)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
52.4	36.7	7/10
	41.9	4/5
	46.3	8/9
	52.4	1
	65.4	5/4

Table 4 (v)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
56.5	42.3	3/4
	56.5	1
	70.4	5/4
	70.7	5/4
	98.7	7/4

Table 4 (vi)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
58.2	36.0	Phi
	38.7	2/3
	46.3	4/5
	48.6	5/6
	52.4	9/10
	58.2	1
	58.3	1
	104.7	9/5

Table 4 (vii)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
63.7	31.9	1/2
	38.1	3/5
	38.4	3/5
	42.3	2/3
	47.8	3/4
	48.0	3/4
	51.2	4/5
	53.4	5/6
	56.5	8/9
	63.7	1
	96.0	3/2

Table 4 (viii)

Measurement Length (cm)	Peak length (cm)	Fraction of Base Measurement length
80.8	31.9	2/5
	40.5	1/2
	48.6	3/5
	50.0	Phi
	53.8	2/3
	54.0	2/3
	60.8	3/4
	64.5	4/5
	80.8	1

Table 4 (ix)

Tables 4 ((i)-(ix)) show how the peak lengths revealed on Graph 3 can be explained as fractional values of nine base measurement lengths (36.0, 41.2, 46.3, 50.0, 52.4, 56.5, 58.25, 63.7 and 80.8cm) Putting the uncomfortable question as to why would so many measurement units would be used concurrently to one side for the moment, and just following the science, the Stone Circle Internal radii can be expressed as integer multiples of these nine measurement lengths.

Stone Circle	Site Ref (Thom)	Internal Radius Measured from Plans (cm)	Internal Radius expressed as Proposed measurement units	Internal radius match using Proposed measurement units
Shin River	N 2/3	191.0	3x63.7	191.1 (100.1)
Monzie	P 1/13	216.3	6x36.0	216.0 (99.8)
Fountain Hill	B 1/10	252.2	7x36.0	252.0 (99.9)
Shin River	N 2/3	291.2	5x58.2	291.0 (99.9)
Loch Mannoch	G 4/9	301.3	6x50.0	300.0 (99.6)
Esslie Greater	B 2/4	314.0	6x52.4	314.4 (100.1)
Burreldales	B 4/2	315.1	6x52.4	314.4 (99.8)
Miltown of Clava	B 7/2	318.5	5x63.7	318.5 (100.0)
The Mound	N 2/2	328.9	8x41.2	329.6 (100.2)
Shianbank	P 2/8	380.5	6x63.7	382.2 (100.4)
Shianbank	P 2/8	382.3	6x63.7	382.2 (100.0)
Carnousie	B 4/1	392.2	7x56.5	395.5 (100.8)
Blindwells	P 2/3	394.7	11x36.0, 7x56.5	396.0 (100.3), 395.5 (100.2)
Colen	P 2/6	396.0	11x36.0, 7x56.5	396.0 (100.0), 395.5 (99.9)
River Ness	B 7/19	433.0	12x36.0	432.0 (99.8)
Cullerlie	B 2/7	486.0	6x80.8	484.8 (99.8)
Raedykes North	B 3/4	487.2	6x80.8	484.8 (99.5)
Farr Post Office	B 7/17	487.7	6x80.8	484.8 (99.4)
Ardlair	B 1/18	534.1	13x41.2	535.6 (100.3)
Wamphray	G 7/3	539.7	15x36.0	540.0 (100.1)
Dalcross Castle	B 7/6	576.1	16x36.0, 14x41.2, 11x52.4, 9x63.7	576.0 (100.0), 576.8 (100.1), 576.4 (100.1), 573.3 (99.5)
Aviemore	B 7/12	636.6	10x63.7	637.0 (100.0)
Loch Buie	M 2/14	641.0	11x58.2	640.2 (99.9)
Esslie the Lesser	B 2/5	648.0	14x46.3, 13x50.0, 8x80.8	648.2 (100.0), 650.0 (100.3), 646.4 (99.8)
Auchnagorth	B 1/5	650.7	13x50.0	650.0 (99.9)
Tomnagorn	B 2/16	674.7	12x56.5	678.0 (100.5)
Templewood	A 2/8	676.2	12x56.5	678.0 (100.3)

Table 5 (i)

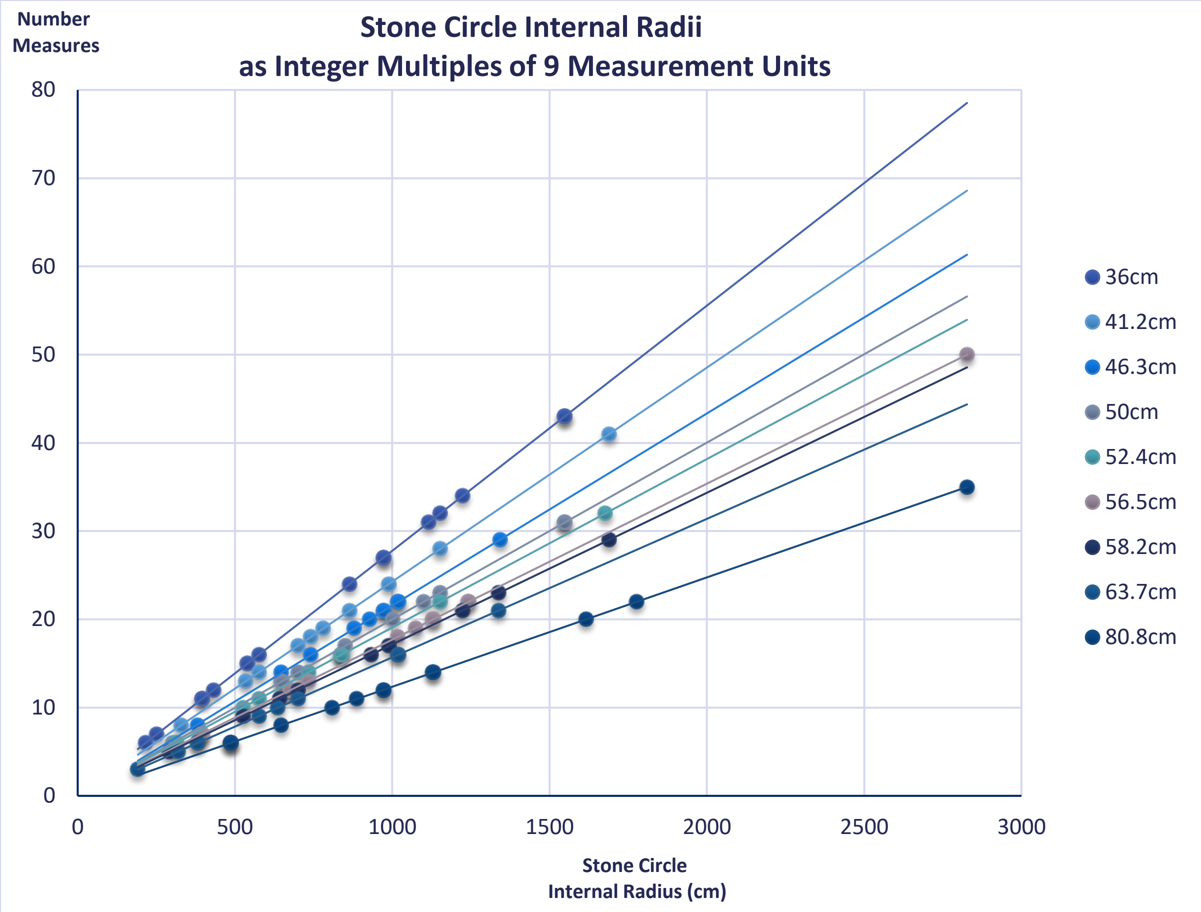
Stone Circle	Site Ref (Thom)	Internal Radius Measured from Plans (cm)	Internal Radius expressed as Proposed measurement units	Internal radius match using Proposed measurement units
Leys of Marlee	P 2/1	699.9	17x41.2, 14x50.0, 12x58.2, 11x63.7	700.4 (100.1), 700.0 (100.0), 698.4 (99.8), 700.0 (100.1)
Westerton	B 1/16	734.0	14x52.4, 13x56.5	733.6 (99.9), 734.5 (100.1)
Aquorthies Kingausie	B 3/1	741.6	18x41.2, 16x46.3	741.6 (100.0), 740.8 (99.9)
Sheldon of Bourtie	B 1/8	781.3	19x41.2	782.8 (100.2)
Loanhead of Daviot	B 1/26	809.0	10x80.8	808.0 (99.9)
Clune Wood	B 3/7	837.7	16x52.4	838.4 (100.1)
Midmar	B 2/17	837.8	16x52.4	838.4 (100.1)
Tomnaverie	B 2/9	838.4	16x52.4	838.4 (100.0)
Miltown of Clava	B 7/2	850.8	17x50.0	850.0 (99.9)
Yonder Bognie	B 1/23	864.3	24x36.0, 21x41.2	864.0 (100.0), 865.2 (100.1)
Esslie the Greater	B 2/4	880.0	19x46.3	879.7 (100.0)
Tyrebagger	B 2/1	888.0	11x80.8	888.8 (100.1)
Easter Aquorthies	B 1/6	928.0	20x46.3	926.0 (99.8)
Moyness	B 6/2	932.0	16x58.2	931.2 (99.9)
Loch Nell	A 1/2	972.9	27x36.0, 21x46.3, 12x80.8	972.0 (99.9), 972.3 (99.9), 969.6 (99.7)
Castle Fraser	B 2/3	973.3	27x36.0, 21x46.3, 12x80.8	972.0 (99.9), 972.3 (99.9), 969.6 (99.6)
West Farr	B 7/16	989.6	24x41.2, 17x58.2	988.8 (99.9), 989.4 (100.0)
Loanhead of Daviot	B 1/26	1001.4	20x50.0	1000.0 (99.9)
Little Urchany	B 6/1	1017.8	22x46.3, 18x56.5, 16x63.7	1018.6 (100.1), 1017.0 (99.9), 1019.2 (100.1)
River Ness	B 7/19	1020.0	22x46.3, 16x63.7	1018.6 (99.9), 1019.2 (99.9)
Tomnagorn	B2/16	1075.0	19x56.5	1073.5 (99.9)
Tarland	B 2/8	1100.0	22x50.0	1100.0 (100.0)

Table 5 (ii)

Stone Circle	Site Ref (Thom)	Internal Radius Measured from Plans (cm)	Internal Radius expressed as Proposed measurement units	Internal radius match using Proposed measurement units
Aquorthies Kingausie	B 3/1	1115.1	31x36.0	1116.0 (100.1)
Druid Temple	B 7/18	1129.3	20x56.5, 14x80.8	1130.0 (100.1), 1131.2 (100.2)
Aviemore	B 7/12	1131.0	20x56.5, 14x80.8	1130.0 (99.9), 1131.2 (100.0)
Esslie the Greater	B 2/4	1152.0	32x36.0, 28x41.2, 23x50.0, 22x52.4	1152.0 (100.0), 1153.6 (100.1), 1150.0 (99.8), 1152.8 (100.1)
Mains of Gask	B 7/15	1223.0	34x36.0, 21x58.2	1224.0 (100.1), 1222.2 (99.9)
Sunhoney	B 2/2	1241.8	22x56.5	1243.0 (100.1)
Carnousie	B 4/1	1242.9	22x56.5	1243.0 (100.0)
Drannandow	G 4/3	1338.2	23x58.2, 21x63.7	1338.6 (100.0), 1337.7 (100.0)
Millton	B 4/4	1344.0	29x46.3	1342.7 (99.9)
Clava South	B 7/1	1547.8	43x36.0, 31x50.0	1548.0 (100.0), 1550.0 (100.1)
Clava Middle	B 7/1	1548.5	43x36.0, 31x50.0	1548.0 (100.0), 1550.0 (100.1)
Urquhart	B 5/1	1616.6	20x80.8	1616.0 (100.0)
Sheldon of Bourtie	B 1/8	1677.7	32x52.4	1676.8 (99.9)
West Farr	B 7/16	1689.7	41x41.2, 29x58.2	1689.2 (100.0), 1687.8 (99.9)
Mains of Gask	B 7/15	1777.0	22x80.8	1777.6 (100.0)
Latheron Wheel	N 1/13	2827.0	50x56.5, 35x80.8	2825.0 (99.9), 2828.0 (100.0)

Table 5 (iii) Dimensions of the stone circles expressed as multiples of the proposed measurement lengths. The mean match of closeness of internal radii measured as measurement lengths to the measured internal radii from the plans is 99.98% +/- 0.2.

It is also interesting that of the 66 rings, 25 of these share their internal radius with at least one other circle. There are six rings that share their internal radius with two other circles. This high level of common dimensions explains the step-like pattern of the curve observed in Graph 2 for the internal radius measurements of the stone circles.



Graph 3 The sixty-six stone circle rings all appear to have internal radii that can be explained by expressing their lengths as a multiple integer of one of the nine proposed measurement units. The next question is whether the nine measurement lengths were individual unrelated lengths or formed a part of a measurement system.

Relationship between Proposed Measurement Units

Given that the method of analysis was designed to avoid using archaic measures from other known ancient cultures as a starting point to account for the dimensions of the Scottish stone circles, it is interesting that we find that the measurement units revealed by the analysis appear to contain two measurement units that are familiar to us from Ancient Egypt, namely the Royal Cubit (52.4cm) and the Sacred Cubit (63.7cm). But had we been only looking for one of these measurements then their significance would have been masked and diluted by the presence of the other eight measurement units. Even more surprisingly one measurement appears to be related to the present-day metre (50.0cm). Whilst the metre may be considered as a modern measurement it is interesting that it is central to obtaining the two ancient Egyptian measures being related to each other through circular geometry

$$63.66 \times \pi = 50.00 \times 4 \quad \text{and} \quad 50 \times \pi = 52.36 \times 3$$

In other words, a circle of diameter 50.0cm has a circumference of three Royal Cubits (3 x 52.36cm) and a circle of one Sacred cubit (63.66cm) diameter has a circumference of two meters (or 4 x 50.0cm).

Further analysis reveals that remarkably all nine measurement lengths are similarly related to one another through circular geometry involving the factor Pi (π =3.142).

Radius	Circumference
63.66	8 x 50.00
50.00	6 x 52.36
52.36	8 x 41.12
46.35	5 x 58.25
80.90	9 x 56.48
35.96	4 x 56.48

Table 6 shows the circular relationships through the factor Pi between the measurement lengths

Furthermore, some lengths are related through the factor Phi (ϕ = 1.618)

50.00 x Phi = 80.9	
50.00 x 3/(2 Phi) = 46.35	

The nine measurement lengths therefore appear to be part of an integrated system that relates each measurement unit to each other through circular geometry. This relationship goes some way in helping us to accept the idea that using so many measurement lengths was not necessarily such a chaotic idea but the reason why so many lengths, despite their mathematical relationship, may have been used remains to be explained. Given the apparent importance of the factor pi in the relationships between the measurement lengths, stone circles whose radii were measured using an integer multiple of one of the measurement lengths will result in a circle whose circumference can be expressed as an integer multiple of one of the other measurement units. The circumferences of the stone circles can be analysed to determine whether they were also planned or whether their lengths simply resulted from the chosen radius because of the pi relationships that exist between the proposed measurement lengths.

Analysis of Stone Circle Circumferences

The measured internal radii of the 66 stone rings can be converted to circumference lengths by multiplying the length of radius by 2π. These circumferences can then be divided by integer values in the same manner as with the analysis of the radii to ascertain whether the same measurements are involved in the dimensions of the stone circle circumferences. Whilst we have seen that the measurement lengths in many instances are related to each other through the factor pi and therefore often will naturally result in a circumference length that can be described in terms of a multiple of one of the other lengths, if the radius length comprises a unit length that is not related to another measurement length through the factor pi, then if the circumference can be described as a multiple of one of the measurement lengths, including the same unit used to describe the radial length then this can be considered as significant and suggests that both the radial length and circumference length were deliberately chosen by the stone circle builders.

As an example, the measurement lengths 35.96cm, 46.35cm, 50.00cm, 52.36cm, 63.66cm and 80.9cm if used to measure the radial length of the stone circle will result in a circumference that can be described as a multiple of one of the other measurement lengths, 56.48cm, 58.25cm, 52.36cm and 41.12cm respectively. However, if the radius comprises a multiple of 41.12cm or 56.48cm for instance, this will only result in a circumference that can be described in terms of a multiple of one of the measurement lengths under special limited circumstances.

The number of peaks obtained from the analysis of the stone circle circumferences can be reduced by applying less stringent limitations in order to maximise the chance of revealing any additional measurement units to the nine already identified from the analysis of the stone circle radii. The graph of frequency of occurrence against measurement length again results in a curve consisting of multiple peaks (Appendix).

By applying the conditions that the frequency of occurrence of a measurement length for the stone circle circumference division is greater than the equivalent mean frequency for the random circle analysis and that the frequency is greater than six, the number of peaks can be reduced to about 180 (10.6% of the data). The same nine measurement lengths found to account for the radius measurements of the sixty-six stone rings also occur as peaks in the circumference graph. By dividing the lengths corresponding to these peaks by the nine measurement lengths most of the lengths can be accounted for by the nine measuring lengths and fractional multiples of these lengths. There are however other lengths that appear to relate to one or more other measurement units that were not apparently associated with the lengths of radii found for the same stone circles.

Peak Lengths (cm) for F_c>6 and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
30.3	32	1.10	-
30.5	31	1.07	-
30.7	32	1.07	41.1x3/4, 46.3x2/3
30.9	28	1.03	41.1x3/4, 46.3x2/3, 50xϕ
31.0	36	1.13	41.1x3/4, 46.3x2/3
31.6	29	1.02	52.4x3/5, 63.6x1/2
32.0	31	1.07	63.6x1/2
32.4	31	1.02	46.3x7/10, 52.4xϕ, 80.9x2/5
32.6	30	1.09	46.3x7/10, 80.9x2/5
32.9	30	1.05	41.1x4/5
33.6	29	1.05	50.0x2/3, 56.5x3/5
34.2	30	1.09	56.5x3/5
34.4	28	1.11	-
34.6	29	1.07	46.3x3/4, 52.4x2/3
35.3	32	1.14	50.0x7/10
35.6	27	1.02	50.0x5/7, 56.5x7/11
35.8	30	1.12	36.0x1
36.4	31	1.27	52.4x7/10
36.6	27	1.09	52.4x7/10
36.8	28	1.16	52.4x7/10
37.3	26	1.04	46.3x4/5, 50.0x3/4, 56.5x2/3
37.7	30	1.23	50.0x3/4, 56.5x2/3
38.1	25	1.13	63.6x3/5
38.4	23	1.01	63.6x3/5
38.7	22	1.05	58.2x2/3
39.4	24	1.03	52.4x3/4, 56.5x7/10, 63.6xϕ
40.0	26	1.23	50.0x4/5
40.2	24	1.18	50.0x4/5, 80.9x1/2
40.7	22	1.04	58.2x7/10, 80.9x1/2
40.8	21	1.01	52.4x7/9, 58.2x7/10, 80.9x1/2
41.1	22	1.01	41.1x1

Table 7 (i)

Peak Lengths (cm) for F_c>6 and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
41.1	22	1.01	41.1x1
41.3	21	1.15	41.1x1
41.8	20	1.03	46.3x9/10, 52.4x45
41.9	21	1.06	46.3x9/10, 52.4x4/5
42.7	21	1.30	56.5x3/4, 63.6x2/3
42.9	23	1.14	50.0x6/7
43.1	22	1.18	36.0x6/5
43.5	21	1.12	52.4x5/6
43.6	19	1.03	58.2x3/4
43.8	21	1.08	58.2x3/4
44.0	20	1.09	58.2x3/4
44.4	23	1.29	63.6x7/10
44.6	20	1.19	63.6x7/10
44.9	18	1.11	36.0x5/4, 50.0x9/10, 63.6x7/10
45.2	17	1.01	36.0x5/4, 50.0x9/10, 56.5x4/5
45.3	19	1.18	56.5x4/5
45.7	18	1.03	-
45.9	17	1.06	-
46.3	19	1.25	46.3x1
46.4	17	1.07	46.3x1, 58.2x4/5
46.6	16	1.03	46.3x1, 58.2x4/5
46.8	18	1.07	58.2x4/5
47.0	19	1.14	52.4x9/10
47.1	16	1.09	52.4x9/10
47.4	16	1.07	52.4x9/10
47.7	19	1.26	63.6x3/4
47.9	19	1.28	36.0x4/3, 63.6x3/4
48.0	18	1.15	36.0x4/3, 63.6x3/4
48.5	16	1.12	80.9x3/5
48.6	17	1.16	80.9x3/5
49.1	15	1.09	41.1x5/4
49.3	15	1.03	41.1x5/4

Table (ii)

Peak Lengths (cm) for F_c>6 and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
49.4	16	1.14	41.1x5/4
50.1	21	1.46	50.0x1
50.3	18	1.34	36.0x7/5
50.7	14	1.10	56.5x9/10, 63.6x4/5
50.8	16	1.13	56.5x9/10, 63.6x4/5
51.3	14	1.12	41.1x5/4, 63.6x4/5
51.6	15	1.24	41.1x5/4
52.4	17	1.22	52.4x1, 58.2x9/10
52.6	12	1.02	52.4x1, 58.2x9/10
52.8	15	1.21	63.6x5/6
53.2	13	1.26	63.6x5/6
53.6	15	1.15	80.9x2/3
53.8	13	1.01	36.0x3/2, 80.9x2/3
54.2	13	1.14	36.0x3/2, 80.9x2/3
54.6	13	1.03	41.1x4/3
54.7	13	1.16	41.1x4/3
55.0	12	1.10	41.1x4/3
55.3	15	1.17	46.3x6/5
55.6	15	1.39	46.3x6/5
55.9	13	1.19	46.3x6/5
56.1	14	1.43	-
56.5	13	1.21	56.5x1, 63.6x8/9, 80.9x7/10
56.6	16	1.42	56.5x1, 63.6x8/9, 80.9x7/10
56.7	12	1.09	56.5x1, 63.6x8/9, 80.9x7/10
57.1	15	1.34	63.6x9/10
57.2	15	1.21	63.6x9/10
57.7	11	1.28	36.0x8/5, 41.1x7/5
57.8	15	1.16	36.0x8/5, 41.1x7/5, 46.3x5/4
58.2	12	1.04	46.3x5/4, 58.2x1
58.4	13	1.34	58.2x1
58.7	11	1.10	41.1x10/7

Table 7 (iii)

Peak Lengths (cm) for F_c>6 and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
59.0	9	1.06	-
59.1	14	1.39	50.0x13/11
59.6	9	1.13	46.3x9/7
59.9	14	1.39	36.0x5/3, 50.0x6/5
60.0	12	1.13	36.0x5/3, 50.0x6/5
60.1	11	1.11	36.0x5/3, 50.0x6/5
60.5	14	1.21	80.9x3/4
60.9	13	1.34	80.9x3/4
61.2	12	1.21	-
61.6	10	1.10	41.1x3/2
62.0	13	1.16	41.1x3/2, 46.3x4/3
62.3	9	1.27	50.0x5/4
62.4	9	1.10	50.0x5/4
62.8	10	1.23	52.4x6/5
63.0	10	1.03	36.0x7/4, 52.4x6/5
63.2	10	1.52	36.0x7/4
63.5	11	1.26	63.6x1
63.6	11	1.21	63.6x1
63.9	10	1.20	63.6x1
64.0	10	1.37	-
64.6	10	1.33	36.0x9/5, 80.9x4/5
65.0	9	1.25	36.0x9/5, 46.3x7/5, 80.9x4/5
65.1	9	1.25	46.3x7/5, 80.9x4/5
65.2	8	1.04	52.4x5/4, 80.9x4/5
65.3	8	1.03	52.4x5/4
65.5	9	1.03	52.4x5/4
65.6	9	1.29	41.1x8/5
66.6	11	1.38	41.1x8/5, 50.0x4/3
66.7	12	1.46	50.0x4/3
67.0	10	1.09	46.3x13/9
67.1	7	1.15	46.3x13/9, 52.4x9/7

Table 7 (iv)

Peak Lengths (cm) for $F_c>6$ and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
67.3	8	1.08	56.5x18/11, 52.4x9/7
67.8	12	1.46	56.5x6/5, 58.2x7/6
67.9	7	1.01	56.5x6/5, 58.2x7/6
68.0	11	1.53	56.5x6/5, 58.2x7/6
68.3	9	1.22	41.1x5/3
68.7	10	1.25	41.1x5/3
68.9	8	1.29	-
69.1	9	1.13	56.5x11/9, 63.6x13/12, 80.9x6/7
69.5	10	1.69	46.3x3/2
69.9	7	1.03	46.3x3/2, 50.0x7/5, 53.4x4/3, 58.2x6/5
70.4	8	1.14	56.5x5/4
70.6	7	1.08	56.5x5/4
71.0	10	1.37	50.0x17/12
71.1	12	1.67	58.2x11/9
71.5	8	1.36	50.0x10/7
71.6	8	1.36	50.0x10/7, 80.9x8/9
71.8	T8	1.19	36.0x2, 41.1x7/4
72.6	9	1.32	36.0x2, 58.2x5/4, 80.9x9/10
72.8	8	1.27	58.2x5/4, 80.9x9/10
73.1	7	1.25	52.4x7/5, 58.2x5/4, 80.9x9/10
73.2	8	1.31	52.4x7/5, 80.9x9/10
73.4	7	1.15	52.4x7/5
73.7	10	2.04	56.3x13/10
74.4	8	1.07	46.3x8/5
74.7	8	1.38	52.4x10/7, 58.2x9/7
74.8	8	1.36	50.0x3/2
75.2	7	1.09	50.0x3/2, 56.5x4/3
75.4	10	1.54	56.5x4/3
75.6	7	1.15	56.5x4/3
75.8	7	1.30	52.4x13/9, 58.2x13/10
76.4	10	1.61	63.6x6/5

Table 7 (v)

Peak Lengths (cm) for F_c>6 and (F_c / F_{cm} >1)	Frequency of Occurrence (F_c)	Ratio Frequency (F_c)/Frequency Random Mean (F_{cm})	Lengths expressed as proposed Measurement Units
77.0	9	1.70	46.3x5/3
77.5	7	1.40	46.3x5/3, 58.2x4/3
78.7	7	1.52	52.4x3/2
78.9	7	1.13	56.5x7/5
79.4	7	1.25	56.5x7/5, 63.6x5/4
79.8	7	1.32	50.0x8/5, 63.6x5/4
79.9	7	1.08	50.0x8/5, 63.6x5/4
80.0	10	1.56	50.0x8/5
80.4	10	2.22	-
80.9	7	1.46	36.0x9/4, 46.3x7/4, 50/ϕ, 80.9x1
81.3	7	1.08	46.3x7/4, 58.2x7/5, 80.9x1
82.4	7	1.30	41.1x2
82.6	7	1.56	41.1x2
83.3	9	2.31	46.3x9/5, 50.0x5/3
84.7	8	1.74	52.4/ ϕ, 56.5x3/2, 63.6x4/3
84.9	7	1.27	56.5x3/2, 63.6x4/3
85.4	7	1.79	52.4x18/11
86.1	8	1.82	46.3x6/7
86.2	10	2.44	36.0x12/5
87.5	7	1.40	50.0x7/4, 58.2x3/2
87.6	10	2.63	50.0x7/4, 58.2x3/2
88.6	7	1.49	50.0x16/9
89.8	9	2.43	36.0x5/2, 50.0x9/5
90.9	7	1.71	63.6x10/7
92.6	9	2.65	41.1x9/4, 46.3x2
96.0	7	1.94	36.0x8/3, 41.1x7/3, 52.4x11/6
97.3	7	2.12	52.4x13/7, 58.2x5/3
98.7	8	2.35	36.0x11/4, 41.1x12/5, 56.5x7/4
100.1	8	1.90	50.0x2
113.1	10	4.35	36.0xπ, 41.1x11/4, 56.5x2, 80.9x7/5
116.5	8	2.86	50.0x9/3, 58.2x2

Table 7 (vi)

The vast majority of the lengths can be expressed as simple fractions and multiples of the nine measurement units proposed from the radial analysis of the stone circles. The fractions described are those that when multiplied by the unit measure result in a length that is within 0.5% of the peak length. Some of the peak lengths cannot be accurately described as fractional values of one of the measurement units and have been marked with a dash (-). These lengths are tabulated below and when divided by each other show that they can all be described in terms of another possible base measurement unit, namely 45.7cm.

Peak Length (cm)	Lengths expressed as Fractional Multiple of 45.7cm
30.3	45.7 x 2/3
30.5	45.7 x 2/3
34.4	45.7 x 3/4
45.7	45.7 x 1
45.9	45.7 x 1
56.1	45.7 x 11/9
59.0	45.7 x 9/7
61.2	45.7 x 4/3
64.0	45.7 7/5
68.9	45.7 x 3/2
80.4	45.7 x 7/4

Table 8

The length 45.7cm is suggested to be the tenth measurement unit giving us the following units;

36.0, 41.1, 45.7, 46.3, 50.0, 52.4, 56.5, 58.2, 63.6 and 80.9cm

The fact that a circle of 58.25cm radius has a circumference of 8 x 45.75cm is consistent with the other measurement units all being connected through circular geometry. It is interesting that a circle of radius 58.25cm has a circumference of 366cm and that this could be considered as the number of nights in a year. It is worth considering that the circle in prehistoric times may have been divided into 366 "Megalithic degrees" rather than the 360 degrees used today consistent with the number of nights in a year.

It is also interesting that whilst we have discovered a 50.0cm measure representing a half a metre, 45.7cm represents almost exactly half a yard. Four of the measurement lengths revealed by the analysis can therefore be considered as known measurement units, the Royal Cubit (52.36cm), the Sacred Cubit (63.66cm), the Half Metre (50.00cm) and the Half Yard (45.75cm). The fact that these measures relate to stone circles constructed around 3000BC gives a surprisingly early origin for these measures and a surprising possible northern origin for these measurement lengths.

The ten measurement units revealed from the division of internal radius and circumference of the 66 stone circles can be described as a series of circles as shown in Diagram 3.

The frequency of occurrence of 116.5cm is more than half that of the proposed 58.25cm measure. This might imply that the 116.5cm length is actually used as a measurement length in its own right. Graph 2 also shows that a length around 161.8cm (161.9-162.0) occurs as often as the 80.9cm length suggesting that this length also may have been used as a measurement length in its own right. Otherwise it would be expected to occur only half as frequently as its half-length base measure. If this is correct then we have a situation where rather than discovering one megalithic ruler we have uncovered as many as twelve megalithic rulers which are all related to each other through circular geometry.

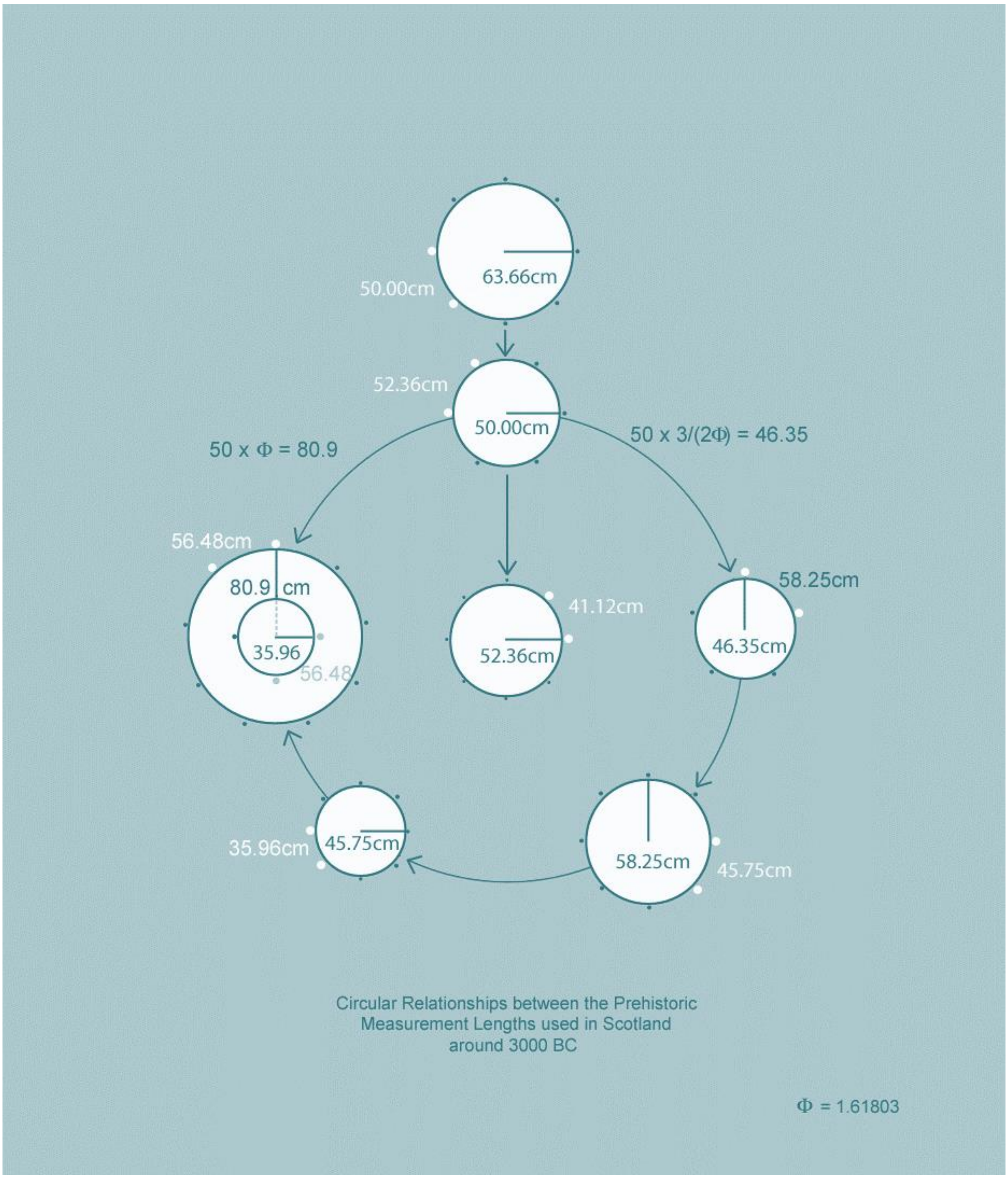


Diagram 3 shows the circular relationships between the measurement lengths 35.96, 41.12, 45.75, 46.35, 50.00, 52.36, 56.48, 58.28, 63.66 and 80.9cm. There are two additional measurement lengths, 116.5cm and 161.8cm representing twice the length of the 58.25cm and 80.9cm lengths, which can be considered as the diameters of the circles of radii 58.25cm and 80.9cm.

Radius Length (cm)	Circumference Length (cm)
35.96	4 x 56.48
46.35	5 x 58.25
50.00	6 x 52.36
52.36	8 x 41.12
58.25	8 x 45.75
63.66	8 x 50.00
80.9	9 x 56.48

The Scottish stone circles appear to have been constructed using a system of measures based on twelve measurement units. Both the internal radii and circumferences of the stone circles appear to consist of integer multiples of the measures. Whilst the relationships between the measurement lengths, where one measure used to form the radius results in a circumference consisting of a multiple of one of the other measures, would therefore naturally result in a stone circle circumference consisting of multiples of a different measure, this is not always the case where certain measurement lengths such as 56.48cm and 41.12cm which only appear as arcs of circumference in Diagram 3. In the cases where 56.48cm and 41.12cm lengths have been used to measure the radius of stone circles, sometimes multiples of seven units were used so that the circumference can be expressed as a multiple of 44 times the radial length perhaps in these cases the close practical approximation of 22/7 as representing pi was chosen by our prehistoric ancestors to achieve these relationships. This suggests that the length of the circumference was important to the design of the stone circles. Table 10(i) shows examples of circles whose radii and circumferences can be described as multiples of the same measurement length using the 56.48cm and 41.12cm unit length. Other stone circles that use the 41.12cm and 56.48cm length to form a radius result in circumferences where measures laid as arcs on the circumference represent integer values of megalithic degrees of arc (Table 10 (ii)).

Circle	Radius	Circumference
Dalcross	14 x 41.12	88 x 41.12
Yonder Bognie	21 x 41.12	132 x 41.12
Esslie the Greater	28 x 41.12	176 x 41.12
Carnousie	7 x 56.48	44 x 56.48
Blindwells	7 x 56.48	44 x 56.48
Colen	7 x 56.48	44 x 56.48

Table 10 (i)

Circle	Radius	Arc Length
The Mound	8 x 41.12	56.48cm = 10 MD
Ardlair	13 x 41.12	46.35cm = 5MD
Aquorthies Kingausie	18 x 41.12	63.66cm = 5MD
Sheldon of Bourtie	19 x 41.12	80.9cm = 6MD
West Farr	24 x 41.12	50cm = 3MD
West Farr	41 x 41.12	58.25cm = 2MD
Carnousie	7 x 56.48	41.12cm = 6MD
Blindwells	7 x 56.48	41.12cm = 6MD
Colen	7 x 56.48	41.12cm = 6MD
Tomnagorn	12 x 56.48	46.35cm = 4MD 58.25cm = 5MD
Templewood	12 x 56.48	46.35cm = 4MD 58.25cm = 5MD
Westerton	13 x 56.48	50.00cm= 4MD
Little Urchany	18 x 56.48	52.36cm = 3MD
Tomnagorn	19 x 56.48	56.48cm = 3MD
Druid Temple	20 x 56.48	58.25cm = 3MD
Sunhoney	22 x 56.48	63.66cm = 3MD
Carnousie	22 x 56.48	63.66cm = 3MD

Table 10 (ii)

These findings suggest that the circumferences of the stone circles were also important and that apart from the choice of most measures as the radius automatically resulting in a circumference consisting of an integer multiple of another measure, sometimes certain integer multiples of lengths of radius may have been chosen to give circumferences that either consisted of multiples of the same measure used to measure the radius or resulted in circumferences where certain important arc angles could be described in terms of one of the other measurements as an arc length.

Dimensions of Stone Circles expressed in terms of the Proposed Measurement Lengths

The stone circle dimensions can be described as multiple lengths of the measurement units revealed from the analysis both for the length of radius and the length of circumference.

Stone Circle	Site Ref (Thom)	Internal Radius (cm)	Radius as measurement units (R)	Circumference Calculated from R (cm)	Circumference as measurement units
Shin River	N 2/3	191.0	3x63.7	1200.0	24x50.0
Monzie	P 1/13	216.3	6x36.0	1355.7	33x41.12, 24x56.48
Fountain Hill	B 1/10	252.2	7x36.0	1581.6	44x35.96, 28x56.48
Shin River	N 2/3	291.2	5x58.2	1830.0	40x45.75
Loch Mannoch	G 4/9	301.3	6x50.0	1885.0	36x52.36
Esslie Greater	B 2/4	314.0	6x52.4	1973.9	48x41.12, 31x63.66
Burreldales	B 4/2	315.1	6x52.4	1973.9	48x41.12, 31x63.66
Miltown of Clava	B 7/2	318.5	5x63.7	1999.9	40x50
The Mound	N 2/2	328.9	8x41.2	2066.9	-
Shianbank	P 2/8	380.5	6x63.7	2399.9	48x50.0
Shianbank	P 2/8	382.3	6x63.7	2399.9	48x50.0
Carnousie	B 4/1	392.2	7x56.5	2484.1	44x56.48, 39x63.66
Blindwells	P 2/3	394.7	11x36.0, 7x56.5	2485.4 2484.1	44x56.48, 39x63.66 44x56.48, 39x63.66
Colen	P 2/6	396.0	11x36.0, 7x56.5	2485.4 2484.1	44x56.48, 39x63.66 44x56.48, 39x63.66
River Ness	B 7/19	433.0	12x36.0	2711.3	48x56.48
Cullerlie	B 2/7	486.0	6x80.8	3049.9	61x50.0, 54x56.48
Raedykes North	B 3/4	487.2	6x80.8	3049.9	61x50.0, 54x56.48
Farr Post Office	B 7/17	487.7	6x80.8	3049.9	61x50.0, 54x56.48
Little Urchany	B 6/1	526.0	10x52.4, 9x58.2	3289.9 3294.0	80x41.12, 71x46.35 72x45.75
Ardlair	B 1/18	534.1	13x41.2	3355.7	-
Wamphray	G 7/3	539.7	15x36.0	3389.2	60x56.48
Dalcross Castle	B 7/6	576.1	16x36.0, 14x41.2, 11x52.4, 9x63.7	3615.1 3617.1 3618.9 3599.9	79x45.75, 78x46.35, 69x52.36, 64x56.48 31x161.8 88x41.12, 78x46.35 64x56.48, 31x161.8 88x41.12 72x50.0
Aviemore	B 7/12	636.6	10x63.7	3999.9	80x50.0
Loch Buie	M 2/14	641.0	11x58.2	4026.0	112x36.0, 88x45.75

Table 11(i)

Stone Circle	Site Ref (Thom)	Internal Radius (cm)	Radius as measurement units (R)	Circumference Calculated from R (cm)	Circumference as measurement units
Esslie the Lesser	B 2/5	648.0	14x46.3, 13x50.0, 8x80.8	4077.2 4084.1 4066.5	88x46.35, 70x58.25, 64x63.66, 35x116.5 78x52.36 72x56.48
Auchnagorth	B 1/5	650.7	13x50.0	4084.1	78x52.36
Tomnagorn	B 2/16	674.7	12x56.5	4258.5	-
Templewood	A 2/8	676.2	12x56.5	4258.5	-
Leys of Marlee	P 2/1	700.0	17x41.2, 14x50.0, 12x58.2, 11x63.7`	4392.2 4398.2 4391.9 4399.9	96x45.75, 69x63.66 107x41.12, 84x52.36 96x45.75, 69x63.66 107x41.12, 88x50.0 84x52.36
Westerton	B 1/16	734.0	14x52.4, 13x56.5	4605.8 4613.4	112x41.12 57x80.9
Aquorthies Kingausie	B 3/1	741.6	18x41.2, 16x46.3	4650.6 4659.6	93x50.0 89x52.36, 80x58.25 40x116.5
Sheldon of Bourtie	B 1/8	781.3	19x41.2	4908.9	-
Loanhead of Daviot	B 1/26	809.0	10x80.8	5083.1	90x56.48
Clune Wood	B 3/7	837.7	16x52.4	5263.8	128x41.12
Midmar	B 2/17	837.8	16x52.4	5263.8	128x41.12
Tomnaverie	B 2/9	838.4	16x52.4	5263.8	128x41.12
Miltown of Clava	B 7/2	850.8	17x50.0	5340.7	102x52.36, 66x80.9, 33x161.8
Yonder Bognie	B 1/23	864.3	24x36.0, 21x41.2	5422.6 5425.7	117x46.35, 96x56.48 67x80.9 -
Esslie the Greater	B 2/4	880.0	19x46.3	5533.3	98x56.48, 95x58.25
Tyrebagger	B 2/1	888.0	11x80.8	5591.4	136x41.12, 99x56.48, 96x58.25, 48x116.5

Table 11 (ii)

Stone Circle	Site Ref (Thom)	Internal Radius (cm)	Radius as measurement units (R)	Circumference Calculated from R (cm)	Circumference as measurement units
Easter Aquorthies	B 1/6	928.0	20x46.3	5824.5	162x36.0, 100x58.25, 72x80.9, 50x116.5
Moyness	B 6/2	932.0	16x58.2	5855.9	128x45.75, 92x63.66
Loch Nell	A 1/2	972.9	27x36.0, 21x46.3, 12x80.8	6100.5 6115.7 6099.7	122x50.0, 108x58.25 105x63.66 122x50.0, 108x58.25
Castle Fraser	B 2/3	973.3	27x36.0, 21x46.3, 12x80.8	6100.5 6115.7 6099.7	122x50.0, 108x58.25 105x63.66 122x50.0, 108x58.25
West Farr	B 7/16	989.6	24x41.2, 17x58.2	6200.7 6221.9	124x50.0 173x36.0, 136x45.75
Loanhead of Daviot	B 1/26	1001.4	20x50.0	6283.2	120x52.36
Little Urchany	B 6/1	1017.8	22x46.3, 18x56.5, 16x63.7	6407.0 6387.7 6399.8	140x45.75,110x58.25 55x116.5 122x52.36, 79x80.9 178x36.0, 128x50.0
River Ness	B 7/19	1020.0	22x46.3, 16x63.7	6400.1 6399.8	178x36.0, 128x50.0 178x36.0, 128x50.0
Tomnagorn	B2/16	1075.0	19x56.5	6742.6	164x41.12
Tarland	B 2/8	1100.0	22x50.0	6911.5	132x52.36
Aquorthies Kingausie	B 3/1	1115.1	31x36.0	7004.2	124x56.48, 110x63.66
Druid Temple	B 7/18	1129.3	20x56.5, 14x80.8	7097.5 7116.3	142x50.0 126x56.438, 88x80.9
Aviemore	B 7/12	1131.0	20x56.5, 14x80.8	7097.5 7116.3	142x50.0 126x56.48, 88x80.9
Esslie the Greater	B 2/4	1152.0	32x36.0, 28x41.2, 23x50.0, 22x52.4	7230.2 7234.2 7225.7 7237.7	158x45.75,156x46.35 128x56.48 - 128x56.48,124x58.25 62x116.5 176x41.12

Table 11 (iii)

Stone Circle	Site Ref (Thom)	Internal Radius (cm)	Radius as measurement units (R)	Circumference Calculated from R (cm)	Circumference as measurement units
Mains of Gask	B 7/15	1223.0	34x36.0, 21x58.2	7682.1 7685.9	136x56.48, 95x80.9 168x45.75, 95x80.9, 66x116.5
Sunhoney	B 2/2	1241.8	22x56.5	7807.2	134x58.25, 67x116.5
Carnousie	B 4/1	1242.9	22x56.5	7807.2	134x58.25, 67x116.5
Drannandow	G 4/3	1338.2	23x58.2, 21x63.7	8417.9 8399.7	184x45.75,149x56.48 168x50.0
Millton	B 4/4	1344.0	29x46.3	8445.5	145x58.25
Clava South	B 7/1	1547.8	43x36.0, 31x50.0	9715.6 9738.9	172x56.48, 60x161.8 186x52.36,152x63.66
Clava Middle	B 7/1	1548.5	43x36.0, 31x50.0	9715.6 9738.9	172x56.48, 60x161.8 186x52.36
Urquhart	B 5/1	1616.6	20x80.8	10166.2	180x56.48
Sheldon of Bourtie	B 1/8	1677.7	32x52.4	10527.6	256x36.0
West Farr	B 7/16	1689.7	41x41.2, 29x58.2	10592.9 10613.9	- 232x45.75,229x46.35
Mains of Gask	B 7/15	1777.0	22x80.8	11182.8	311x36.0,272x41.12 198x56.48,192x58.25 96x116.5
Latheron Wheel	N 1/13	2827.0	50x56.5, 35x80.8	17743.7 17790.8	- 315x56.48,110x161.8

Table 11 (iv)

Arc Angles represented by the Measurement Lengths that describe the Circumference

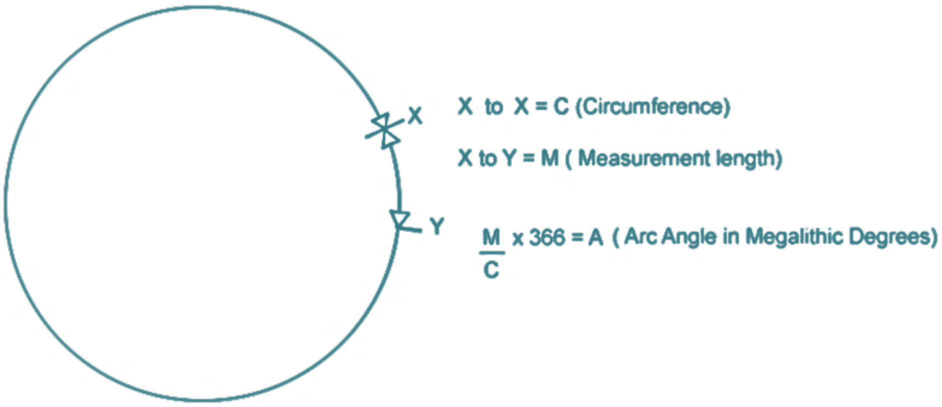
Each of the measurement lengths laid along the circumference of the stone circle describes an angle of arc relative to the centre of the circle. This angle can be determined in terms of Megalithic Degrees by assuming the Neolithic circle comprised of 366 MD or "Megalithic degrees". The arc angle represented by each of the measurement units (35.96, 41.12, 45.75, 46.35, 50.00, 52.36, 56.48, 58.25, 63.66 and 80.9cm) can be calculated using the formula

Arc Angle (A)(MD) **= y x Measurement Length (M)**(cm) **x 366 / Circumference Length (C)**(cm)

Where **A** is measured as megalithic degrees (366 megalithic degrees =360˚)

M is one of the measurement lengths (35.96, 41.12, 45,75, 46.35, 50.0, 52.36, 56.48, 58.25, 63.66 and 80.9cm) and **y** is an integer representing the number of measurement lengths

C is the circumference length calculated from the stone circle internal radius described as **zM** where **z** is an integer (**C=2πzM**)



The arc angles corresponding to multiple lengths of the measurement units can also be calculated to produce tables of angles for each of the stone circle rings. The frequency of occurrence of the different angles can be calculated for the sixty-six stone rings to determine whether the circumference lengths of the circles were designed to enable certain preferred angles to be represented by measurement lengths laid as arcs along the circumference.

The circumference (C) of each of the stone circles is divided by the lengths (M) 35.96cm, 41.12cm, 45.75cm, 46.35cm, 50.00cm, 52.36cm, 56.48cm, 58.25cm, 63.66cm, 80.9cm, 116.5cm and 161.8cm to calculate the arc angle represented by each measurement length laid on the circumference as an arc length.

Table 12 shows the frequency of occurrence of angles measured as megalithic degrees representing multiples of measurement lengths within the range 3 MD - 68 MD at intervals of 0.1 MD for the most frequently occurring arc angles (≥ 40).

Angle (MD)	Frequency	Angle (MD)	Frequency	Angle (MD)	Frequency	Angle (MD)	Frequency
4.1-4.2	45	21.2-21.3	43	34.9-35.0	47	50.9-51.0	63
4.7-4.8	46	21.8-21.9	51	35.3-35.4	42	51.4-51.5	47
5.3-5.4	44	22.8-22.9	54	35.4-35.5	47	51.5-51.6	61
5.4-5.5	41	23.5-23.6	52	36.0-36.1	46	51.7-51.8	40
6.4-6.5	41	23.9-24.0	54	37.0-37.1	78	53.3-53.4	44
8.3-8.4	56	24.2-24.3	40	37.5-37.6	46	53.9-54.0	42
8.5-8.6	41	24.9-25.0	41	37.6-37.7	43	54.0-54.1	72
9.2-9.3	40	25.0-25.1	41	38.8-38.9	64	55.6-55.7	62
9.4-9.5	45	25.4-25.5	54	40.0-40.1	65	56.4-56.5	40
10.1-10.2	41	25.7-25.8	69	41.1-41.2	41	56.5-56.6	47
10.9-11.0	56	25.8-25.9	52	41.6-41.7	45	57.1-57.2	40
11.4-11.5	46	26.1-26.2	41	41.8-41.9	52	58.2-58.3	129
11.9-12.0	47	26.9-27.0	50	41.9-42.0	53	58.9-59.0	42
12.5-12.6	45	27.0-27.1	48	42.3-42.4	45	60.0-60.1	65
13.9-14.0	43	27.8-27.9	48	42.4-42.5	55	61.0-61.1	58
14.5-14.6	59	28.2-28.3	52	43.2-43.3	43	61.1-61.2	40
15.2-15.3	50	29.1-29.2	90	43.6-43.7	60	61.8-61.9	48
15.4-15.5	45	29.6-29.7	44	44.4-44.5	41	62.8-62.9	52
16.4-16.5	53	30.0-30.1	47	44.5-44.6	40	62.9-63.0	49
16.6-16.7	74	30.5-30.6	63	45.0-45.1	59	63.6-63.7	46
16.9-17.0	40	31.4-31.5	67	45.7-45.8	77	64.8-64.9	45
17.1-17.2	64	32.4-32.5	40	46.2-46.3	40	65.5-65.6	40
18.5-18.6	56	32.7-32.8	47	47.1-47.2	62	65.6-65.7	41
18.8-18.9	53	32.8-32.9	40	47.9-48.0	51	65.8-65.9	65
19.4-19.5	50	32.9-33.0	68	48.5-48.6	40	66.6-66.7	46
20.0-20.1	45	33.2-33.3	42	48.6-48.7	44	67.9-68.0	43
20.2-20.3	42	33.3-33.4	53	49.3-49.4	41	68.6-68.7	44
20.9-21.0	54	33.9-34.0	41	49.9-50.0	48		
21.1-21.2	43	34.3-34.4	55	50.0-50.1	54		

Table 12

Many of the most frequently occurring angles are those that correspond to the measurement lengths themselves but in Megalithic degrees, this phenomenon is an artefact of the division process and can be ignored for our purposes though some of the lengths may coincidentally coincide with important angles such as 41.12cm and 41MD. 58.2-58.3MD appears as the most commonly found angle due to its association as a radius length of a circle whose circumference is 366.0cm however this may also be important in relation to 2 x 29MD.

The number of angles for consideration can be further reduced by only considering angles that are integer values within plus or minus 0.1 MD. Table 13 shows integer angles that occur more than forty times.

Angle (MD)	Frequency	Composition
4	45	1x4MD
10	41	1x10MD
11	56	1x11MD
12	47	4x3MD
14	43	**14 MD**
17	104	**17 MD**
20	45	2x10MD
21	97	7x3MD
22	51	2x11MD
23	54	**23 MD**
24	54	8x3MD
25	82	5x5MD
26	93	**2 x 13 MD**
27	98	**9x3MD/27MD**
28	48	2 x 14 MD
29	90	**29 MD**
30	47	3x10MD
33	108	11x3MD
34	41	**2 x 17MD**
35	47	7x5MD
36	46	12x3MD
37	78	**37 MD**
39	64	**3 x 13 MD**
40	65	4x10MD
42	105	14x3MD
45	59	15x3MD
48	51	16x3MD
50	54	5x10MD
51	63	**3x17MD**
54	72	18x3MD/ **2x27MD**
59	42	-
60	65	6x10MD/20x3MD
61	98	**(366/6)**
62	48	
63	101	21x3MD
66	65	22x3MD
68	43	**4 x 17 MD**

Table 13

Many of the most commonly occurring angles appear to be multiples of 3MD and 5MD. It is interesting that many of the circles may have been designed to allow a measurement length used as an arc on the circumference to represent multiples of 3MD, 5MD and 10MD of angle. This association between measurement lengths and arc angles suggests that the circles may have been involved in some way with measuring the alignment of heavenly bodies and following their apparent movement along arcs on the circle's circumference. When the remaining most frequently occurring arc angles are examined a pattern emerges where multiples of 7, 13, and 17 MD account for 7 of the 14 angles.

Angle (MD)	Frequency	Composition
14	43	2x7MD
17	64	**1x17MD**
23	54	**1x23MD**
26	52	2x**13MD**
27	50	**1x27MD**
28	48	4x7MD
29	90	**1x29MD**
34	41	2x**17MD**
37	78	**1x37MD**
39	64	3x**13MD**
59	42	1x59MD
61	58	61MD = 1/6 x366MD
62	48	1x62MD
68	43	4x**17MD**

Table 14

As far as the other arc angles are concerned, an angle of 61MD describes one sixth of the circumference of a circle consisting of 366 Megalithic Degrees and its significance may lie in the ease with which this angle can be obtained simply by dividing the circumference into 6 equal parts using the radius length to create arcs centred on the circumference. More importantly the arc angles 13, 17, 23, (27), 29 and 37 are hypotenuse lengths on right angle triangles known as Pythagorean triplets, where each side of the triangle can be described as an integer value; these angles and multiples of these angles account for 9 of the 14 arc angles. Interestingly these numbers are also prime numbers but most importantly the occurrence of these numbers gives us a clue as to the importance of triangles formed of sides of integer units to the stone circle builders. A rope or string consisting of equally spaced knots could have been used to form right angle triangles such as the 5:12:13, 8:15:17, 13:19:23, 20:21:29 and the 12:35:37 right angle triangles and even the 17, 21, 27.018 triangle that is not quite a perfect Pythagorean triplet. Perhaps these triangles were useful in describing a North, South, East, West reference frame on the ground having determined true South on the ground using the alignment of the Sun when the length of shadow of a pole is shortest at the Sun's azimuth when the Sun is due South in the sky. Perhaps these Pythagorean Triplet hypotenuse numbers had a special significance to the stone circle builders and were used as the foundation of their ingenious measurement system.

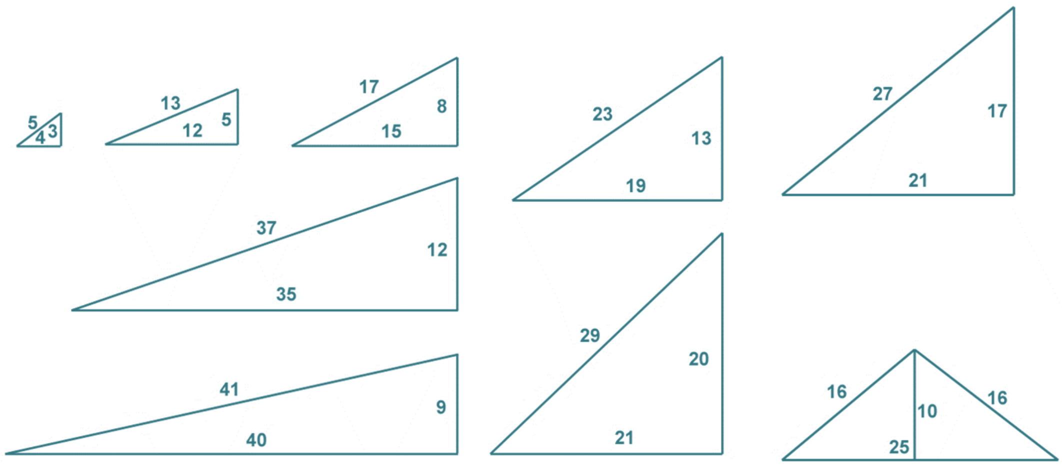


Diagram 5 Pythagorean Triplet Right-angle Triangles, a near triplet (17,21,27.018) and an isosceles triangle (16,16,25) whose integer hypotenuse lengths coincide with important numbers associated with the prehistoric measurement system.

The idea that a measurement length could be placed on the circumference of a circle as an arc to describe a particular angle is intriguing in that if this was the intention then there is the suggestion that the calibration of the circumference into arcs and arc lengths may have had a role in marking the passage of time. The position and movement of the Sun and stars along the horizon with the passing days could be followed relative to the arc lengths laid out on the circle's circumference. In other words the circle could form the basis of a calendrical device where the apparent movement of the Sun, stars and Moon along the horizon could be followed as they rise and set at different positions along the eastern and western arcs of the horizon at different times of the year as the Earth rotates about its axis each day as it orbits the Sun during its annual cycle described through the four seasons.

The possibility of measurement lengths used as arcs on a circle's circumference marking angles of rotation or time raises the further intriguing possibility that the measurement lengths apart from corresponding to arc angles may have measured time intervals in another way, namely when the measurement lengths were used as pendulums.

Measurement Lengths used as Pendulums

The possibility of measurement lengths being used to measure both physical length on the ground and time by using the same measurement length as a pendulum can be examined by determining the periods of the measurement lengths when used as pendulums. The period of oscillation of a pendulum can be calculated using the formula

$$\mathbf{T = 2\,\pi \sqrt{(l / g)}}$$

where **T** is the period (in seconds (s)) of the pendulum describing the time it takes for a pendulum to swing back and fore from its original starting position back to that almost same position (allowing for friction). **l** is the length of the pendulum in metres measured from the point of suspension of the pendulum to the centre of gravity of the bob and **g** is the acceleration due to gravity in m/s^2.

The lengths of the pendulums (**l**) tested are those measurement lengths that appear from the analysis of the Scottish stone circle radii and circumferences, namely 0.3596m, 0.4112m, 0.4575m, 0.4635m, 0.5000m, 0.5236m, 0.5648m, 0.5825m, 0.6366m and 0.809m and the possible double lengths 1.165m and 1.618m. The variation in acceleration due to gravity in the British Isles varies from $9.8116 m/s^2$ at a latitude of 51 degrees in the South of England to $9.8192 m/s^2$ in the Shetland Islands which straddle the 60 degree line of latitude in the far North of Scotland. The International Gravity Formula allows for variation of gravity with distance from the equator. Two effects are accounted for in the formula, namely the centripetal acceleration caused by the Earth's rotation and the oblateness or degree of swelling of the Earth at the equator. The Normal Gravity can be calculated for the latitude 57 degrees North corresponding to Aberdeenshire where the majority of stone circles are located (Diagram 2) using the International Gravity Formula

$$\mathbf{g_0 = 9.7803267714\{1+0.001932185138639 \sin^2\lambda /(1-0.00669437999013 \sin^2 \lambda)^{1/2}\}}$$

where g_0 is referred to as the theoretical gravity or normal gravity and λ is the geographical latitude in degrees. The value calculated for normal gravity for Aberdeenshire is $9.817003 m/s^2$.

Table 15 gives the number of swings or oscillations of pendulums of the different measurement lengths required for the rotation of the Earth by angles measured in megalithic degrees in the range of 1-41 Megalithic Degr

Angle Megalithic Degrees	Pendulum lengths (cm)											
	35.95	41.12	45.75 (Solar)	46.35	50.00 (Solar)	52.36	56.48	58.25	63.66	80.9	116.5	161.8 (Solar)
	Number of swings required per angle of rotation relative to sidereal time (solar time indicated in brackets)											
1 Sidereal (Solar)	391.6	366.2	347.2 (348.1)	344.9	332.1 (333.0)	324.5	312.5	307.7	294.3	261.1	217.6	(185.1)
2	783.2	732.4	(696.2)	689.8	(666.0)	649.0	624.9	615.3	588.6	522.1	435.1	(370.2)
3	1174.7	1098.6	(1044.3)	1034.7	(999.0)	973.5	937.4	923.0	882.9	783.2	652.7	(555.3)
4	1566.3	1464.8	(1392.4)	1379.6	(1331.9)	1298.0	1249.8	1230.7	1177.2	1044.3	870.2	(740.4)
5	1957.9	1830.9	(1740.5)	1724.6	(1664.9)	1622.6	1562.3	1538.3	1471.5	1305.4	1087.8	(925.5)
6	2349.5	2197.1	(2088.7)	2069.5	(1997.9)	1947.1	1874.7	1846.0	1765.8	1566.4	1305.3	(1110.6)
7	2741.1	2563.3	(2436.8)	2414.4	(2330.9)	2271.6	2187.2	2153.7	2060.1	1827.5	1522.9	(1295.7)
8	3132.6	2929.5	(2784.9)	2759.3	(2663.9)	2596.1	2499.6	2461.4	2354.4	2088.6	1740.4	(1480.9)
9	3524.2	3295.7	(3133.0)	3104.2	(2996.9)	2920.6	2812.1	2769.0	2648.7	2349.6	1958.0	(1666.0)
10	3915.8	3661.9	(3481.1)	3449.1	(3329.9)	3245.1	3124.5	3076.7	2943.1	2610.7	2175.6	(1851.1)
11	4307.4	4028.1	(3829.2)	3794.0	(3662.8)	3569.6	3437.0	3384.4	3237.4	2871.8	2393.1	(2036.2)
12	4699.0	4394.3	(4177.3)	4138.9	(3995.8)	3894.1	3749.4	3692.0	3531.7	3132.8	2610.7	(2221.3)
13	5090.6	4760.5	(4525.4)	4483.8	(4328.8)	4218.7	4061.9	3999.7	3826.0	3393.9	2828.2	(2406.4)
14	5482.1	5126.6	(4873.5)	4828.8	(4661.8)	4543.2	4374.3	4307.4	4120.3	3655.0	3045.8	(2591.5)
15	5873.7	5492.8	(5221.6)	5173.7	(4994.8)	4867.7	4686.8	4615.0	4414.6	3916.1	3263.3	(2776.6)
16	6265.3	5859.0	(5569.7)	5518.6	(5327.8)	5192.2	4999.3	4922.7	4708.9	4177.1	3480.9	(2961.7)
17	6656.9	6225.2	(5917.9)	5863.5	(5660.8)	5516.7	5311.7	5230.4	5003.2	4438.2	3698.4	(3146.8)
18	7048.5	6591.4	(6266.0)	6208.4	(5993.7)	5841.2	5624.2	5538.0	5297.5	4699.3	3916.0	(3331.9)
19	7440.0	6957.6	(6614.1)	6553.3	(6326.7)	6165.7	5936.6	5845.7	5591.8	4960.3	4133.5	(3517.0)
20	7831.6	7323.8	(6962.2)	6898.2	(6659.7)	6490.2	6249.1	6153.4	5886.1	5221.4	4351.1	(3702.1)
21	8223.2	7690.0	(7310.3)	7243.1	(6992.7)	6814.8	6561.5	6461.0	6180.4	5482.5	4568.7	(3887.2)
22	8614.8	8056.2	(7658.4)	7588.0	(7325.7)	7139.3	6874.0	6768.7	6474.7	5743.5	4786.2	(4072.3)
23	9006.4	8422.3	(8006.5)	7933.0	(7658.7)	7463.8	7186.4	7076.4	6769.0	6004.6	5003.8	(4257.4)
24	9397.9	8788.5	(8354.6)	8277.9	(7991.7)	7788.3	7498.9	7384.1	7063.3	6265.7	5221.3	(4442.5)
25	9789.5	9154.7	(8702.7)	8622.8	(8324.7)	8112.8	7811.3	7691.7	7357.6	6426.8	5438.9	(4627.7)
26	10181.1	9520.9	(9050.8)	8967.7	(8657.6)	8437.3	8123.8	7999.4	7651.9	6787.8	5656.4	(4812.8)
27	10572.7	9887.1	(9398.9)	9312.6	(8990.6)	8761.8	8436.2	8307.1	7946.2	7048.9	5874.0	(4997.9)
28	10964.3	10253.3	(9747.1)	9657.5	(9323.6)	9086.3	8748.7	8614.7	8240.5	7310.0	6091.5	(5183.0)
29	11355.9	10619.5	(10095.2)	10002.4	(9656.6)	9410.8	9061.1	8922.4	8534.8	7571.0	6309.1	(5368.1)
30	11747.4	10985.7	(10443.3)	10347.3	(9989.6)	9735.4	9373.6	9230.1	8829.2	7832.1	6526.7	(5553.2)
31	12139.0	11351.9	(10791.4)	10692.2	(10322.6)	10059.9	9686.0	9537.7	9123.5	8093.2	6744.2	(5738.3)
32	12530.6	11718.1	(11139.5)	11037.2	(10655.6)	10384.4	9998.5	9845.4	9417.8	8354.2	6961.8	(5923.4)
33	12922.2	12084.2	(11487.6)	11382.1	(10988.5)	10708.9	10311.0	10153.1	9712.1	8615.3	7179.3	(6108.5)
34	13313.8	12450.4	(11835.7)	11727.0	(11321.5)	11033.4	10623.4	10460.8	10006.4	8876.4	7396.9	(6293.6)
35	13705.3	12816.6	(12183.8)	12071.9	(11654.5)	11357.9	10935.9	10768.4	10300.7	9137.5	7614.4	(6478.7)
36	14096.9	13182.8	(12531.9)	12416.8	(11987.5)	11682.4	11248.3	11076.1	10595.0	9398.5	7832.0	(6663.8)
37	14488.5	13549.0	(12880.0)	12761.7	(12320.5)	12006.9	11560.8	11383.8	10889.3	9659.6	8049.5	(6848.9)
38	14880.1	13915.2	(13228.1)	13106.6	(12653.5)	12331.5	11873.2	11691.4	11183.6	9920.7	8267.1	(7034.0)
39	15271.7	14281.4	(13576.3)	13451.5	(12986.5)	12656.0	12185.7	11999.1	11477.9	10181.7	8484.6	(7219.1)
40	15663.2	14647.6	(13924.4)	13796.4	(13319.4)	12980.5	12498.1	12306.8	11772.2	10442.8	8702.2	(7404.2)
41	16054.8	15013.8	(14272.5)	14141.4	(13652.4)	13305.0	12810.6	12614.4	12066.5	10703.9	8919.8	(7589.4)

Table 15 Number of swings of the measurement lengths when used as pendulums required for the rotation of the Earth by angles measured in Megalithic Degrees.

The measurement lengths used as pendulums give extremely close to whole thousands of swings for the time it takes the Earth to rotate by angles 13, 16, 17, 23, 27, 29, 34, 37 and 41 Megalithic Degrees. The angle values are the same Pythagorean Triplet hypotenuse values that were found from the circumference analysis. This finding is remarkable in that it indicates that the measurement system used by Neolithic man some five thousand years ago integrated the measurement of distance and time in the same single system of measurement units. This finding explains the rationale behind the surprising use of multiple measurement lengths and shows the sophistication of our ancestor's measurement technology in being able to come up with a solution that perfectly integrated the measurement of distance on the ground with the measurement of time. There is another consideration regarding the time intervals associated with the angles of rotation of the Earth, the angles given represent sidereal time or time relating to the relative movement of the Earth and stars. It is however also necessary to consider time measured relative to the Sun or solar time. When the number of swings is calculated for the rotation of the Earth relative to the Sun, the pendulum lengths that gave numbers of swings furthest from whole thousands for sidereal time, (Table 15) namely the 45.75cm and 50.00cm and 161.8cm pendulums give much closer to whole thousand values when solar time is used in the calculations. This suggests that there were two types of pendulum ones that were used at night (35.96cm, 41.12cm, 46.35cm, 52.36cm, 58.25cm, 63.66cm, 80.9cm and 116.5cm) and three that were used by day using the Sun for reference (45.75cm, 50.00cm and 161.8cm). It is apparent that the two long pendulums (116.5cm and 161.8cm)which had lengths of twice that of the 58.25cm and 80.9cm measurement lengths have periods of oscillation that result in 5004swings/23 MD and 4998 swings/ 27MD supporting the idea that these lengths were important measurement lengths in their own right. In addition the 161.8cm long pendulum gives 100 swings for the time it takes for the Earth to rotate an angle equivalent to that represented by the diameter of the Sun (mean between perihelion(101.7) and aphelion(98.4)) and also gives 5000 swings per 27 Megalithic Degrees when used as a solar pendulum. It is useful that the 58.25cm and 80.9cm pendulums are sidereal pendulums and yet when doubled in length give pendulums that can be used to measure solar time. The closeness of the lengths 45.75cm and 46.35cm and the 50.00cm and 52.36cm lengths is explained by the fact that the 45.75cm and 50.0cm pendulums are solar pendulums whilst the 46.35cm and 52.36cm pendulums relate to sidereal time.

Solar Pendulum length (cm)	Angle (MD)	Number of swings (Sidereal Time)	Number of swings (Solar Time)
45.75	23	7985	8006
50.00	27	8966	8991
161.8	27	4984	4998

Calibration of the Pendulum lengths

Integrating the measurement of distance and time in one system has another benefit in that it allows the accurate calibration of lengths using the known period of the length when used as a pendulum. Given that a 35.96 cm pendulum length gives 9000 swings for a 23MD rotation of the Earth relative to the stars all that needs to be done is to find two stars that are separated by an Hour Angle of 23MD and align the westernmost star with a vertical pole from a fixed point and then swing the pendulum and count the number of oscillations until the second star aligns with the same pole, adjusting the length of the string until the pendulum gives the required 9000 swings. Another way in which this could be achieved is by using the Pythagorean triplet 5:12:13 right angle triangle which conveniently has one vertex comprising an angle of 23 MD. A knotted rope could be used to lay out this triangle on the ground with the smallest angle (23MD) pointing North. Three upright straight sticks are pushed into the ground at the vertices and alignments made between the smallest angle vertex, and a star aligned with each of the other two vertices. The smallest angle in this triangle is 23MD and so the triangle provides a very convenient calibration frame for the 35.96cm pendulum without having to know two stars separated by precisely 23MD. As the 45.75 cm pendulum also gives a whole thousand number of swings for 23MD, it too can be calibrated using this system, although as it is a solar pendulum, the alignments need to be with the Sun rather than the stars. Two other pendulums are calibrated using 23MD of rotation with respect to sidereal time, namely the 80.9cm pendulum that requires 6000 swings for 23MD and the 116.5cm pendulum which oscillates 5000 times for a rotation of 23MD. The circular relationships between the different measurement units (Diagram 3) might also be used to obtain the various lengths at least approximately by drawing circles using the calibrated pendulums and dividing their circumferences to obtain other measurement lengths.

A measurement system that allows both the physical measurement of length as a ruler and the measurement of time when used as a pendulum offers an extremely accurate method of calibrating the particular lengths that would allow the owner of the measure to achieve measurement lengths that were extremely accurate by adjusting the length of the pendulum length to give the required number of swings for the time it takes the Earth to rotate by the required angle. This ability to calibrate lengths using the period of oscillation has certain advantages over comparing a length with a standard reference length. Thom himself believed that if the Megalith Yard had been used as a measure, then a standard length would need to exist for other rulers to be measured against. The fact that each pendulum length could be accurately calibrated against the stars and Sun by its owner would allow the measurement units to be very accurately determined and would ensure that measurement lengths were consistent across the whole country.

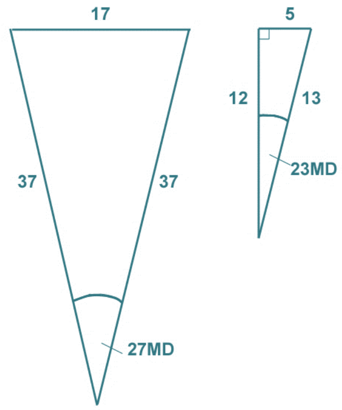


Diagram 6 Triangles that have acute angles of 23 Megalithic degrees and 27 Megalithic degrees at a vertex. Laying these triangles on the ground using an evenly spaced knotted cord and erecting sighting canes at each vertex would allow pendulums to be calibrated by counting the number of oscillations it took for a star to move from its alignment with one stick to the other separated by an arc angle of 23 or 27 Megalithic degrees. These two triangles alone could be used to calibrate six of the pendulums (the 35.96cm, 80.9cm and 116.5cm sidereal pendulums and the 45.75cm, 50.0cm and 161.8cm solar pendulums). Other angles defined in isosceles triangles and arcs laid on the ground using evenly space knotted ropes could similarly be used to obtain the other required angles for calibration.

Angle (Megalithic Degrees)	Pythagorean Right Angle Triangle	Isosceles Triangle	Arc (Radius, arc length)
13		9, 9, 2	
17			24, 7
23	5, 12, 13		
27		37, 37, 17	
29			2, 1
32	11, 18, 21		
34			12, 7
37		8, 8, 5	
41	11, 13, 17		

Table 17 shows the angles required to calibrate the pendulums can be most easily laid out on the ground using a rope with evenly spaced knots along its length to create reference triangles and arcs with canes erected vertically at the vertices for alignment with stars or the Sun.

Calibration of Pendulum Lengths using Star Pairs

From our perspective stars appear to rotate about a central point high in the night sky called the Celestial Pole. Their horizontal position when mapped vertically onto the ground can be described in astronomy as their Hour Angle. The angular separation of two stars can be described in terms of the Hour Angle which is expressed in terms of hours, minutes and seconds separating them. Although the stars move relative to each other over very long time periods they can be considered as separated by constant angles over a period of a century or so for practical purposes. The measurement of the hour angle separating two stars can be achieved by aligning a vertical reference viewing cane with another vertical cane that is aligned with the first star and erecting a second cane that is aligned with a second star from the same viewing cane and measuring the angle described on the ground by the three canes.

Five thousand years ago our ancestors could simply measure the Hour Angle separating stars by using a quadrant to discover star pairs separated by the angles required to calibrate the various pendulums. The quadrant could have easily been formed using an arc from a circle of radius 58.25cm which had a circumference of 366cm where each centimetre represented one megalithic degree. In order for us to rediscover the star pairs our ancestors could have used to calibrate the pendulums an archaeo-astronomy program is required to allow us to see the same night skies that our ancestors saw recognising that the relative position of stars change as a function of time.

The hour angle separating two stars can be calculated from astronomical data by subtraction and then converted into degrees or in this case Megalithic Degrees by dividing the number of seconds by 235.42 seconds (the number of seconds equivalent to one Megalithic Degree for sidereal time). Likewise, for Solar time the Hour Angle can be converted to Megalith Degrees by division by 236.066 seconds.

Identifying star pairs that are separated by angles of 13, 16, 17, 23, 27, 29, 32, 34, 37 and 41 Megalithic Degrees allows the various pendulum lengths to be simply calibrated. This time the western-most star of the star pair is aligned using a pair of canes separated by a short distance, the pendulum is swung and the number of swings counted that are required for the second star to become aligned with the same two canes. The required number for each pendulum can be obtained by lengthening or shortening the pendulum to reduce or increase the pendulum's period.

Considering only the brightest stars in the sky, the following star pairs are examples of those that have been calculated using SkyMap Pro II to have been possible star pairs that could have been used to calibrate pendulums around the year 3000BC.

Star Pair	Constellations	Hour Angle of Separation (s)	Equivalent Megalithic Degrees		Pendulum Length (cm)	Number of Swings
Alnilam Saiph	Orion Orion	1515	6.44 (13/2)		58.25cm	2000
Alnitak Pollux	Orion Gemini	3759	15.97		56.48	5000
Bellatrix Pollux	Orion Gemini	5399	22.93		35.96 80.9 116.5	9000 6000 5000
Capella Betelgeuse	Auriga Orion	5436	23.03 (Solar)		45.75	8000
Arcturus Antares	Bootes Scorpio	6376	<u>**27.00**</u> (Solar)		50.00 161.8	9000 5000
Rigel Procyon	Orion Canis Minor	6868	29.17		46.35	10000
Bellatrix Procyon	Orion Canis Minor	7533	<u>**32.00**</u>		56.48	10000
Deneb Algenib	Cygnus Pegasus	7551	31.99 (Solar)		56.48	10000
Deneb Alpheratz	Cygnus Andromeda (Square of Pegasus)	8004	<u>**34.00**</u>		63.66	10000
Regulus Arcturus	Leo Bootes	17339	2x37		52.36	24000
Capella Mintaka	Auriga Orion	4823	41 x 1/2		41.12	7500

It is interesting that only a handful of constellations are required to find sufficient star pairs in order to accurately calibrate all 12 pendulum lengths and that the stars involved are predominantly the brightest stars in their constellation. Rigel and Betelgeuse are the brightest stars in Orion, whilst Deneb is the brightest star in Cygnus, Pollux the brightest star in Gemini, Capella the brightest star in Auriga, Arcturus the brightest star in Bootes, Antares the brightest star in Scorpio, Procyon the brightest star in Canis Minor, Alpheratz the brightest star in Andromeda and Regulus the brightest star in Leo. Of these brightest stars Pollux, Capella, Arcturus, Procyon, Deneb and Bellatrix are involved in two star pairs. Six of the star pairs involve at least one star from Orion, all of them associated with the main body of the constellation emphasising the importance of this male deity to our ancestors. The separation of some of the star pairs is precisely the angle required for calibration, such as Deneb -Apheratz (34MD), Bellatrix-Procyon (32MD) and Arcturus-Antares (27MD), whilst the others are extremely close and close enough to create an accurate pendulum length. It is interesting to consider the large numbers of oscillations of the pendulum that required to be counted and how this might most easily be achieved by breaking the total number down into groups of hundreds for instance.

The proposal is therefore that the prehistoric measurement system used by Neolithic man in Scotland to measure the stone circles consisted of 12 measurement units 35.96cm, 41.12cm, 45.75cm, 46.35cm, 50.00cm, 52.36cm, 56.48cm, 58.25cm, 63.66cm, 80.9cm, 116.5cm and 161.8cm. The idea of basing a measurement system around a series of pendulums is a philosophically advanced idea and represents a different approach to modern methods of measuring time and distance that require two separate measurement systems and this helps to explain why until now the search for a single ancient measurement length was always doomed to fail. The sophistication of this measurement system may be surprising to us but there is other supporting evidence that this is correct such as the geodesic stone spheres (see Part 3) that were used as long rulers by winding a cord around their flattened circular knobs in prescribed patterns and whose precisely carved dimensions allowed multiple lengths of the proposed measurement lengths to be achieved. Also when the effect of gravitational field is taken into account for the period of a pendulum, in Egypt (see Part 10) it is necessary to shorten the Scottish pendulums to achieve the same period of oscillation and when this is carried out the dimensions of the pyramids of Giza can be revealed as whole hundreds of multiples of the amended Scottish pendulums.

Conclusions

The availability of a large number of accurately drawn plans of Scottish stone circles drawn by Professor Thom allowed the internal radii of the circles to be calculated. When these measured stone circle internal radii were divided by integers a series of lengths were revealed that occur much more frequently than by chance. These lengths are proposed to have been the measurement lengths or rulers used by the stone circle builders to plan and measure the circles. The measurement lengths found consist of twelve different lengths (35.96cm, 41.12cm, 45.75cm, 46.35cm, 50.00cm, 52.36cm, 56.48cm, 58.25cm, 63.66cm, 80.9cm, 116.5cm and 161.8cm) and every length appears to be related to each other through circular geometry where the use of one measurement length as a circle's radius produces a circumference comprising an integer multiple of one of the other lengths. In other words, the length is related through an integer multiple of pi whilst some are related through phi. The twelve measurement lengths therefore provided a prehistoric measurement system that allowed circles to be created that had both radii and circumferences comprising integer multiples of the proposed measurement lengths. The lengths laid out as arcs on the circumference describe angles that can be described as integer values of megalithic degrees (where 366 Megalithic Degrees = 360 Degrees). Furthermore, when the twelve lengths are used as pendulums, they each give whole thousands of swings for the time it takes the Earth to rotate by integer angles of Megalithic Degrees relative to the stars or the Sun. The lengths comprise two types of pendulum, sidereal pendulums for use at night when the movement of the Earth relative to the stars is used as reference and solar pendulums used during the day where the rotation of the Earth relative to the Sun is used for reference. The integer angles are typically those that could be described as the hypotenuse values of Pythagorean triplet triangles, or right-angle triangles where each side is comprised of an integer value. The measurement system used some five thousand years ago in Neolithic Scotland therefore represents a sophisticated metrology that combined the measurement of physical length on the ground and time in the one measurement system and may be considered in many ways to be a more elegant solution to measurement of time and distance to the one we have today especially in its ability to relate our existence on a spinning Earth and our view of the Sun and stars as we spin and orbit the Sun. A highly accurate and relevant prehistoric measurement system was developed where all a person required was a length of string, a bob and the ability to count large numbers and an intimate knowledge of the stars and angles of Pythagorean triplet triangles.

Pendulum Length (cm)	Circular Relationship	Period of Swing	Pendulum Type
35.96	35.96 x π = 2 x 56.48	9000 swings / 23 MD	Sidereal
41.12	41.12 x 4 = 52.36 x π	366 swings /1 MD 15000 swings / 41 MD	Sidereal
45.75	45.75 x 4 = 58.25 x π	8000 swings / 23 MD	Solar
46.35	46.35 x 2 x π = 58.25 x 5	10000 swings / 29 MD	Sidereal
50.00	50.00 x π = 52.36 x 3 50.00/Phi = 80.9 50.00 x 3 x Phi =46.35 x 2	9000 / 27 MD	Solar
52.36	52.36 x π = 41.12 x 4	12000 swings / 37 MD	Sidereal
56.48	56.48 x 9 = 80.9 x 2 x π	5000 swings / 16 MD	Sidereal
58.25	58.25 x 2 x π = 45.75 x 4	4000 swings / 13 MD	Sidereal
63.66	63.66 x π = 50.00 x 4	5000 swings / 17 MD	Sidereal
80.9	80.9 x π x 2 = 56.48 x 9	6000 swings / 23 MD	Sidereal
116.5	116.5 x π = 45.75 x 4	5000 swings / 23 MD	Sidereal
161.8	161.8 x π = 565.48 x 9	5000 swings / 27 MD 100 swings / Sun Diameter	Sidereal/Solar

Bibliography

1. Kendall, D. G. (1974), "Hunting quanta", Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences (276): 231–266

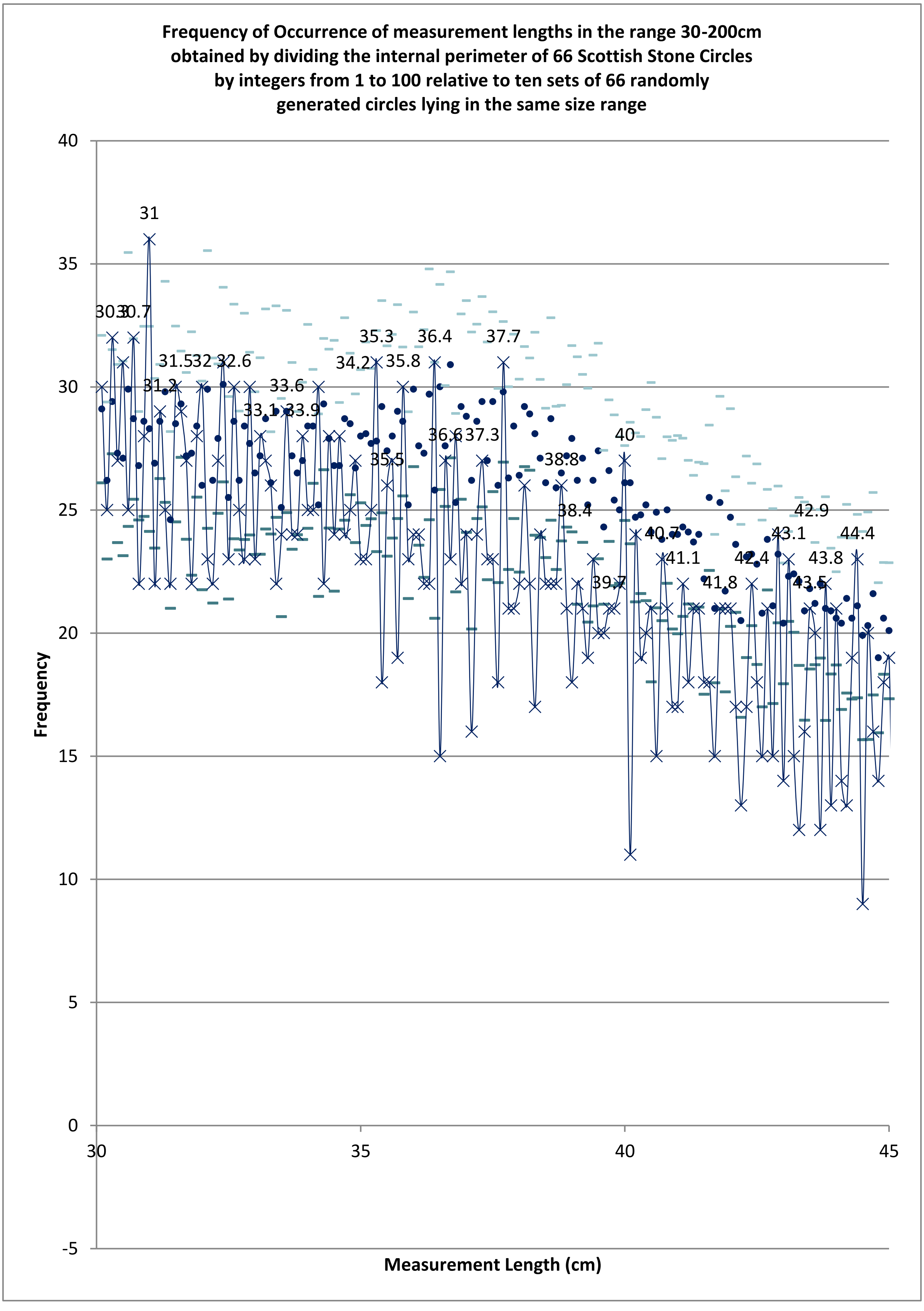
2. Freeman, P. R. (1976), "A Bayesian Analysis of the Megalithic Yard", Journal of the Royal Statistical Society 139 (1)

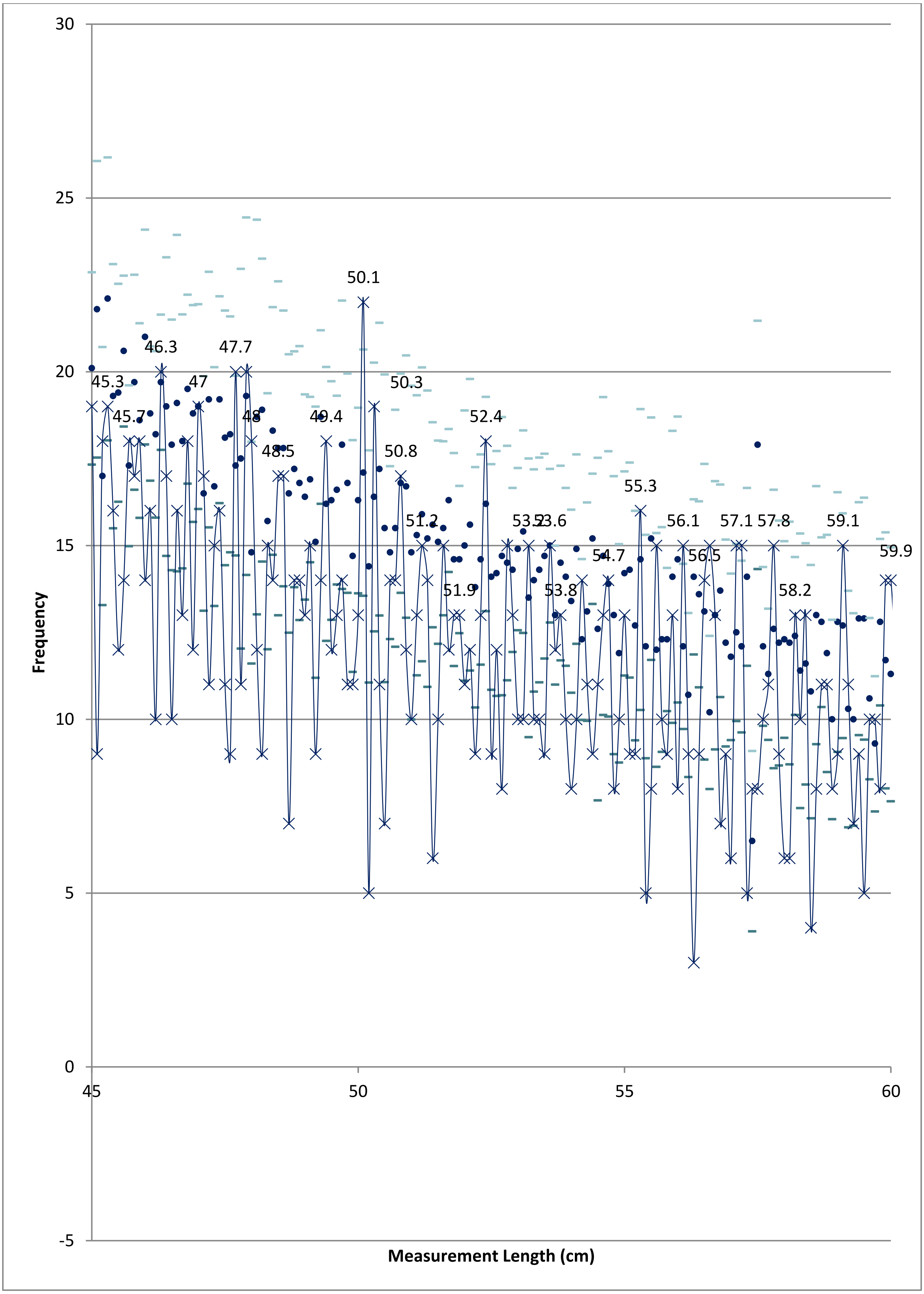
3. Thom, A. (1955), "A Statistical Examination of the Megalithic Sites in Britain", Journal of the Royal Statistical Society. Series A, 118 No.3, 275-295
4. Broadbent, S.R. (1955) Quantum hypothesis, Biometrika, 42, 45-57

5. Thom, A. (1967), “Megalithic Sites in Britain”, ISBN 0198131488

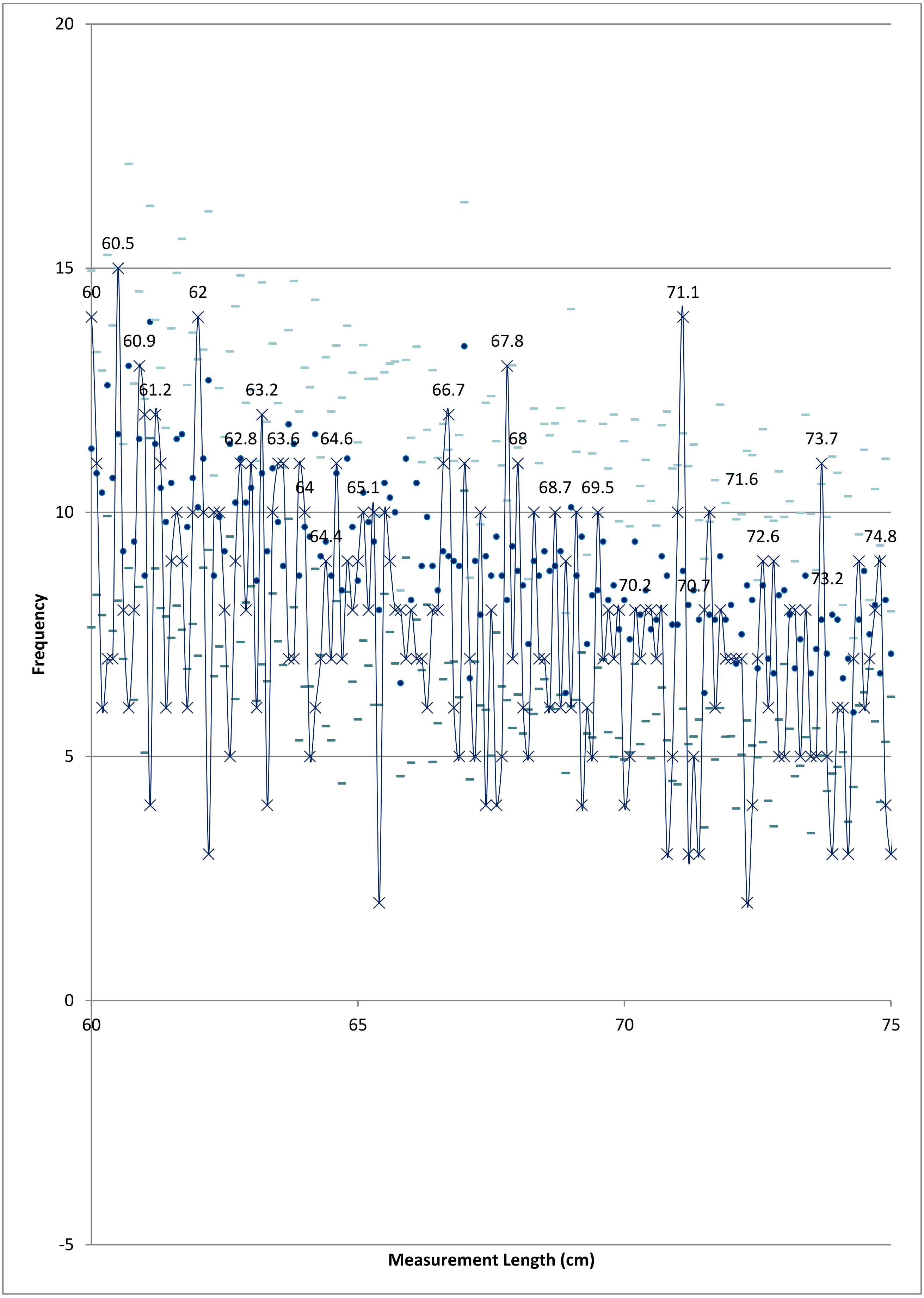
6. Thom, A. and Thom, A.S. collated by Burl, A. (1980), “Megalithic Rings”, British Archaeological Reports (BAR), British Series 81

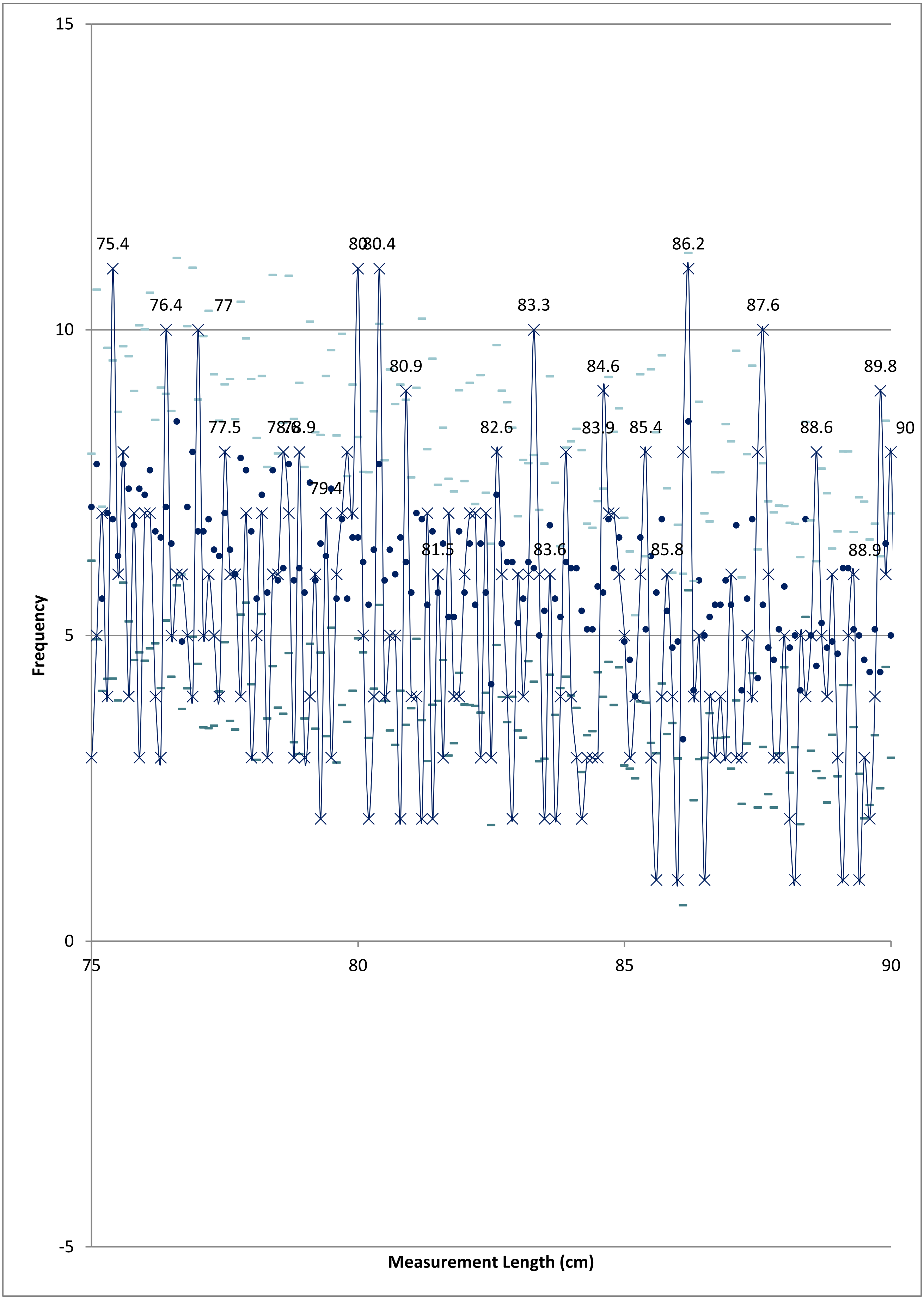
Appendix


Frequency of Occurrence of measurement lengths in the range 30-200cm obtained by dividing the internal perimeter of 66 Scottish Stone Circles by integers from 1 to 100 relative to ten sets of 66 randomly generated circles lying in the same size range
Frequency
Measurement Length (cm)
40
35
30
25
20
15
10
5
0
-5
30
35
40
45
31
30
30.7
31.5
32
32.6
31.2
33.6
33.1
33.9
35.3
36.4
37.7
34.2
35.8
36.6
37.3
35.5
38.8
38.4
40
39.7
40.7
41.1
41.8
42.4
42.9
43.1
43.8
43.5
44.4

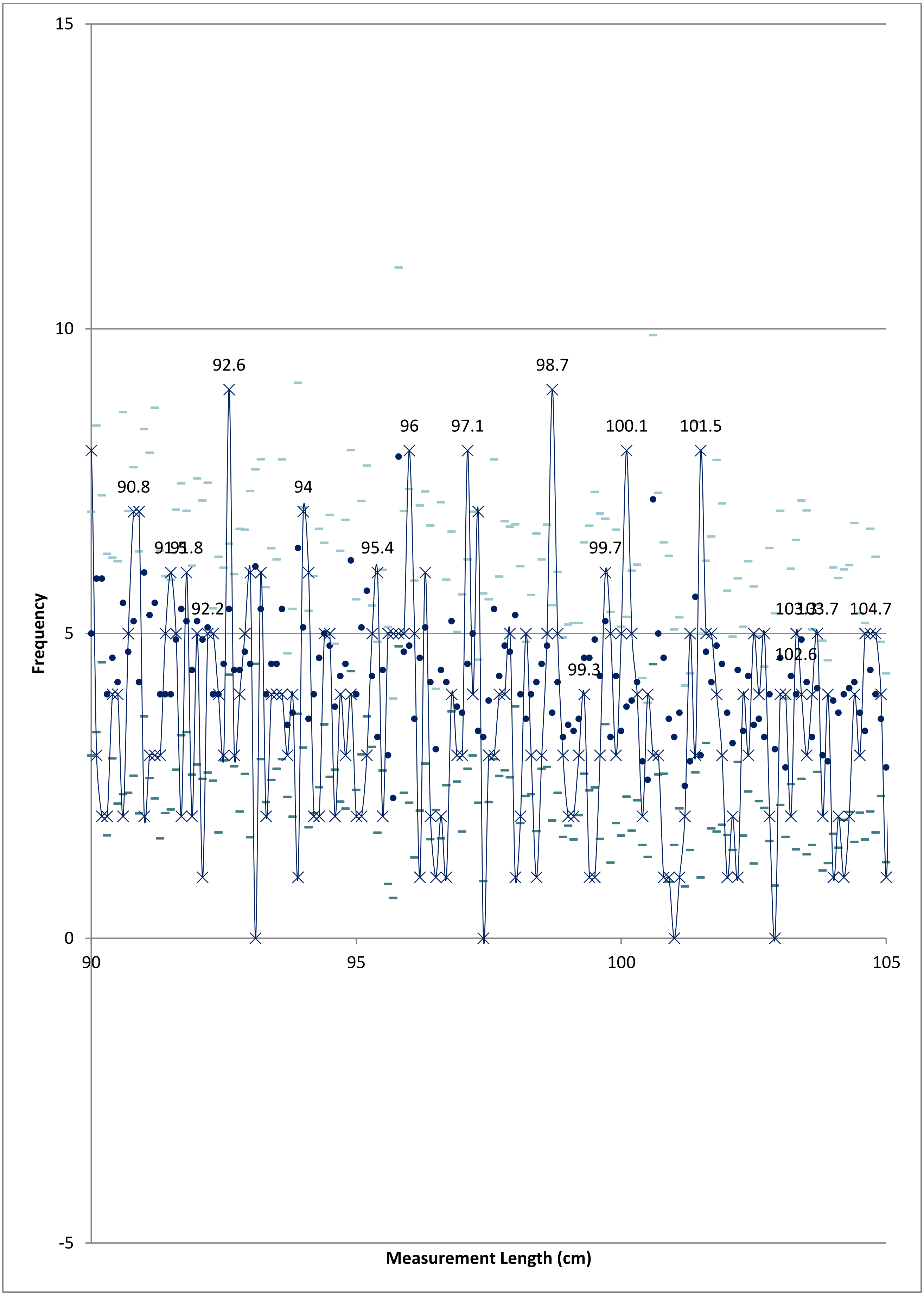


Frequency
Measurement Length (cm)
30
25
20
15
10
5
0
-5
45
50
55
60
50.1
46.3
47.7
45.3
47
50.3
45.7
48
49.4
52.4
48.5
50.8
55.3
51.2
53.5
53.6
56.1
57.1
57.8
59.1
54.7
56.5
59.9
51.9
53.8
58.2

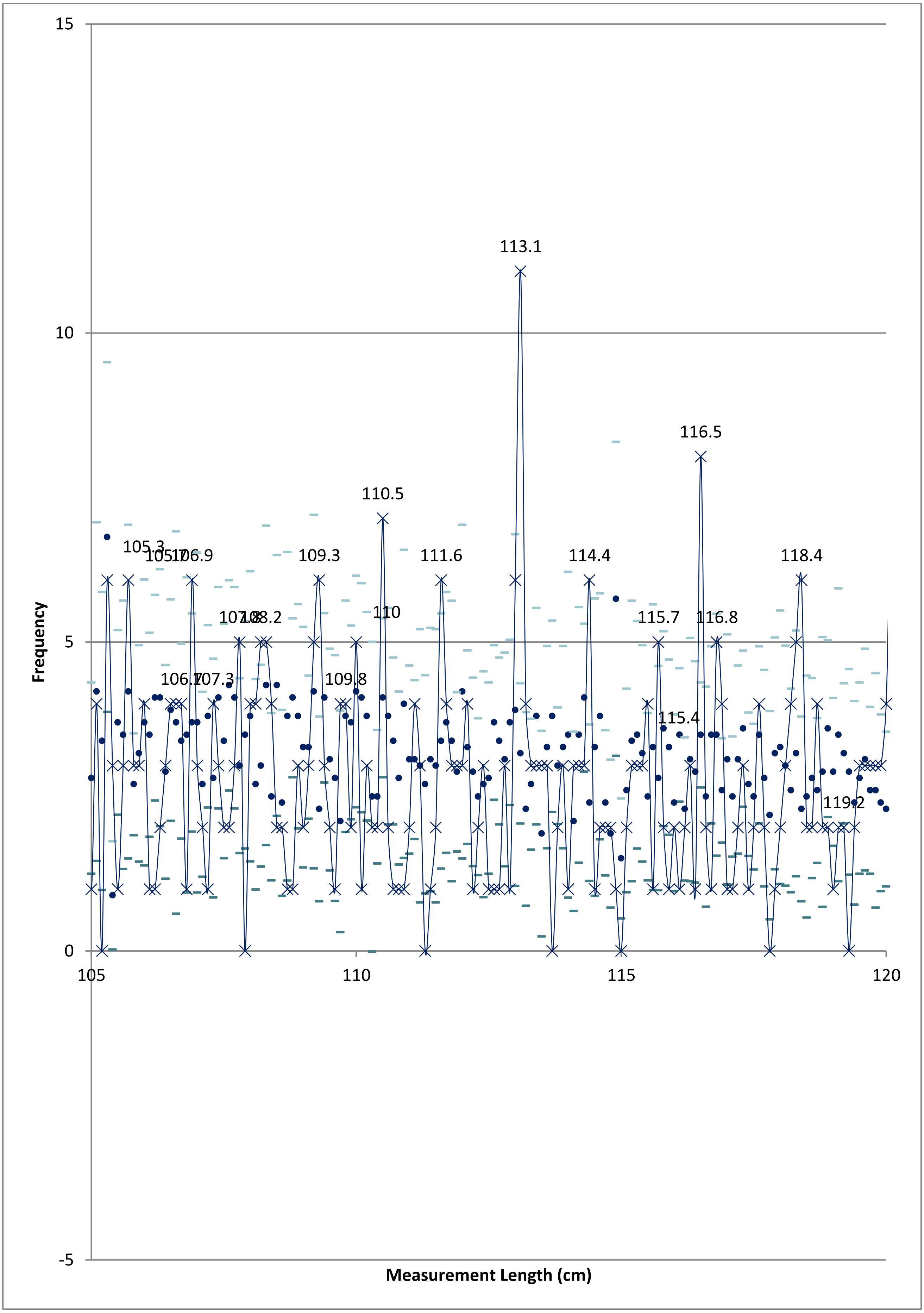

Frequency
Measurement Length (cm)
60.5
60
62
60.9
61.2
63.2
62.8
63.6
64.6
64
65.1
64.4
66.7
67.8
68
68.7
69.5
70.2
70.7
71.1
71.6
72.6
73.7
73.2
74.8

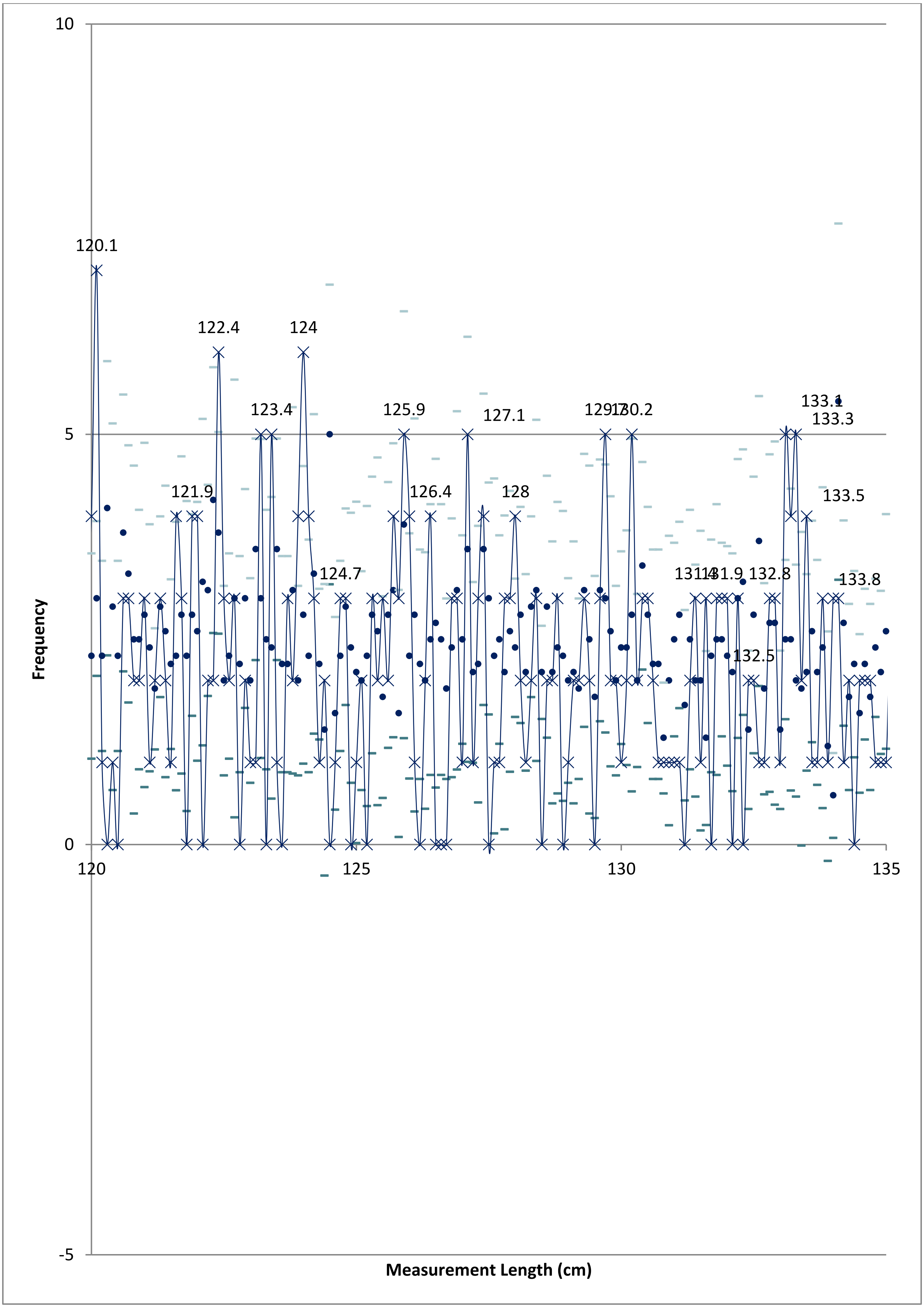


15
10
5
0
-5
75
80
85
90
Frequency
Measurement Length (cm)
75.4
76.4
77
77.5
78.9
79.4
80.4
80.9
81.5
82.6
83.3
83.6
83.9
84.6
85.4
85.8
86.2
87.6
88.6
88.9
89.8

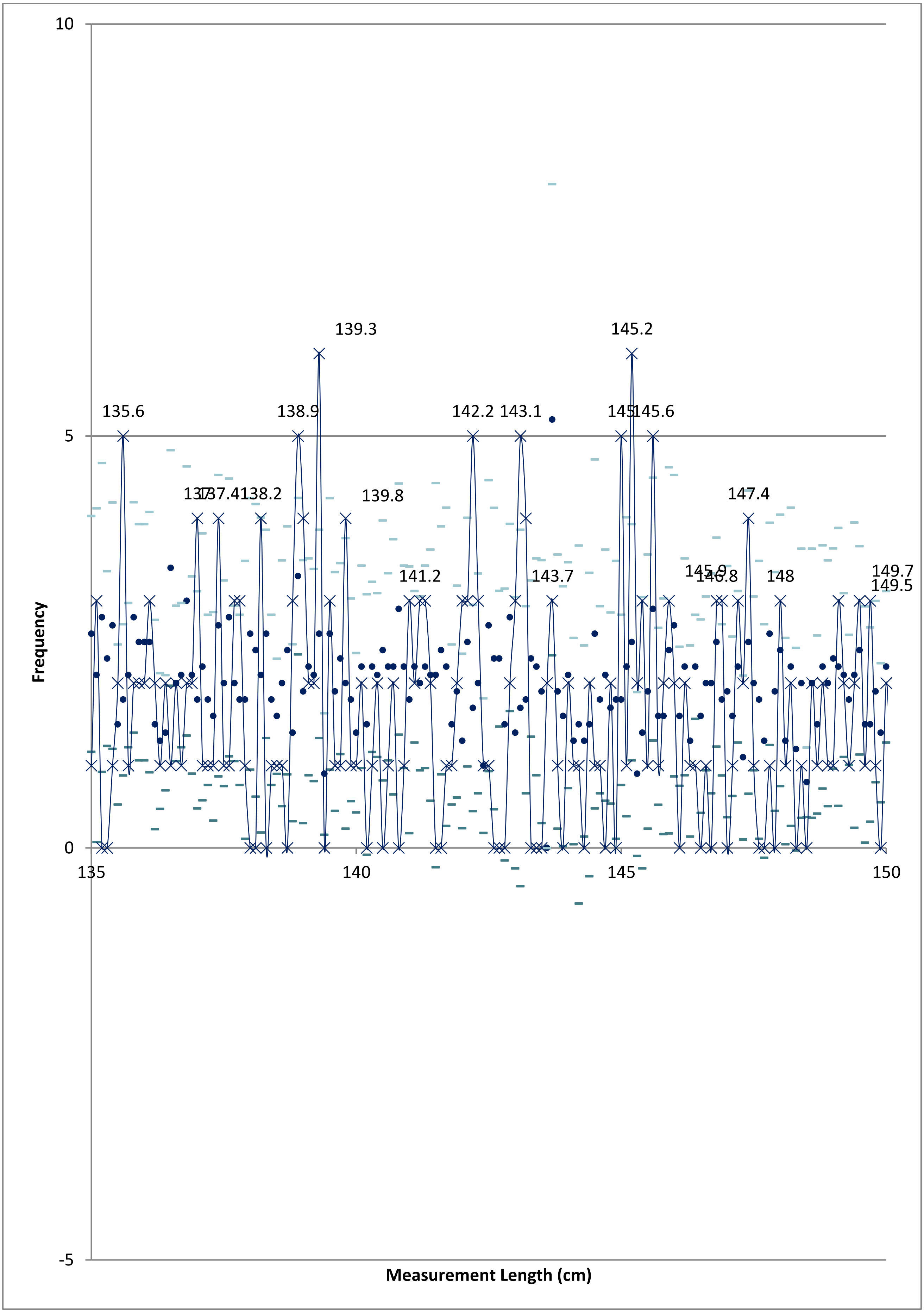


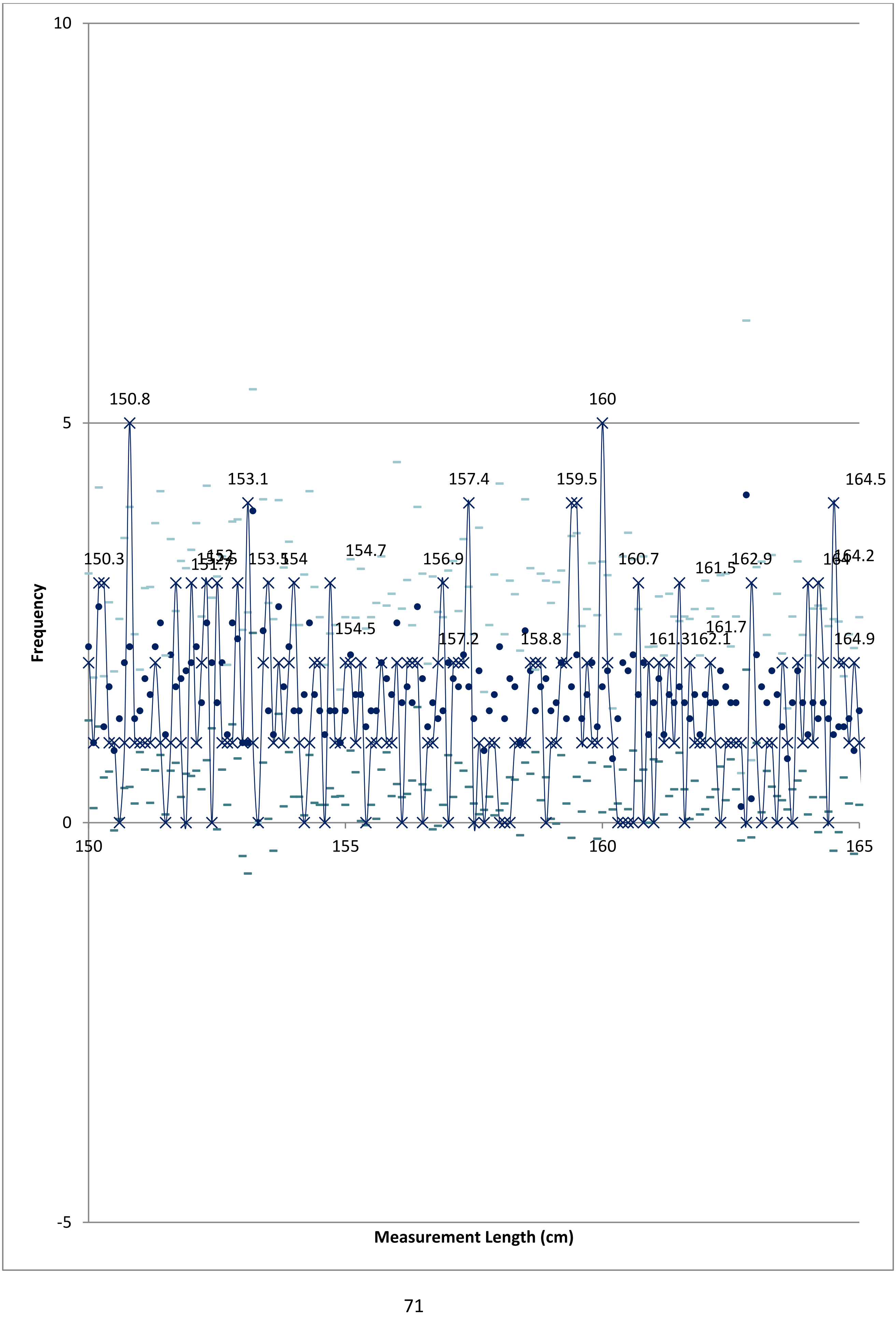
Frequency
Measurement Length (cm)
15
10
5
0
-5
90
95
100
105
92.6
98.7
96
97.1
100.1
101.5
90.8
94
91.5
91.8
95.4
99.7
92.2
103
103.7
104.7
102.6
99.3

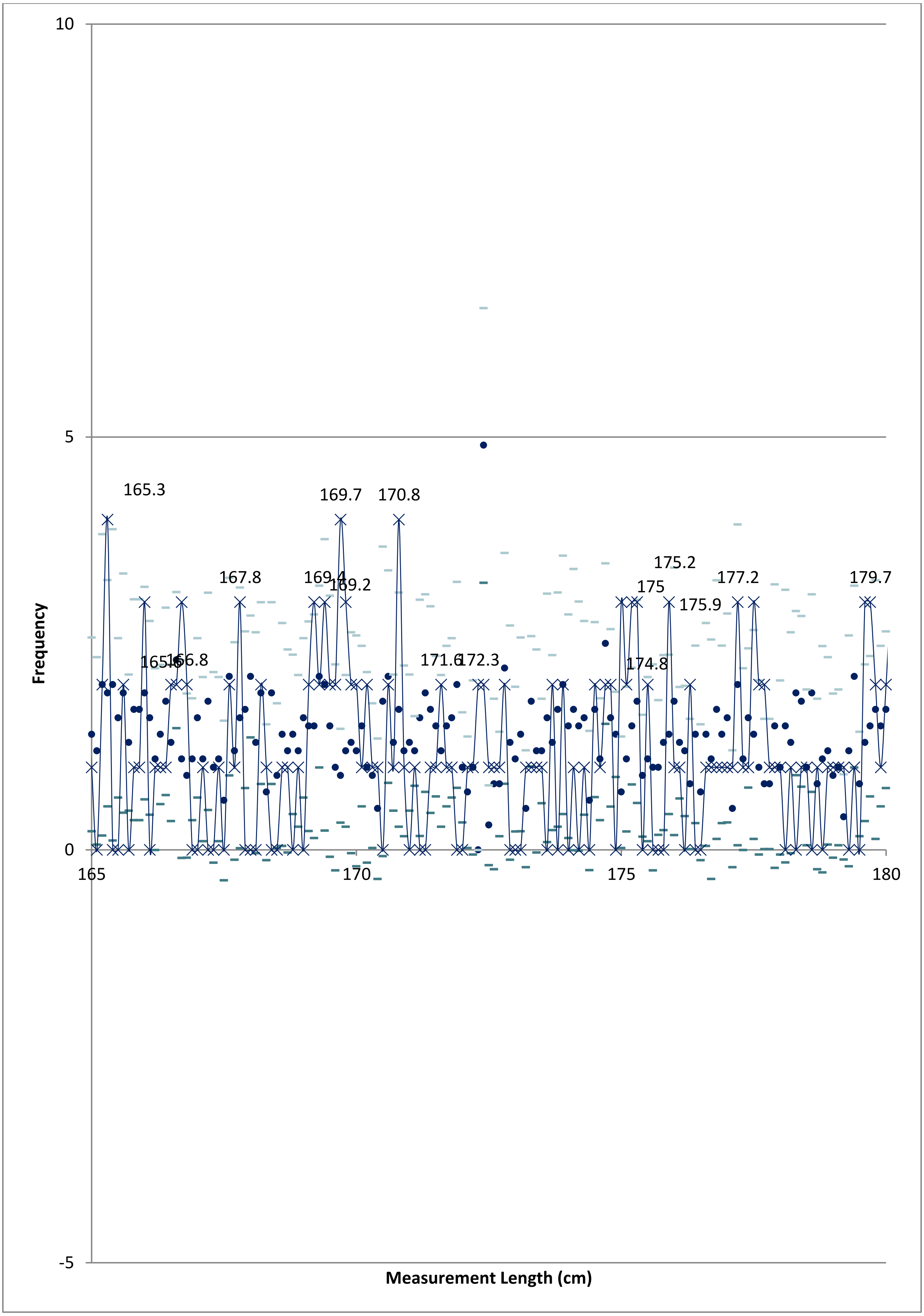

15
10
5
0
-5
105
110
115
120
Frequency
Measurement Length (cm)
113.1
116.5
110.5
105.3
109.3
111.6
114.4
118.4
110
115.7
116.8
109.8
115.4
119.2

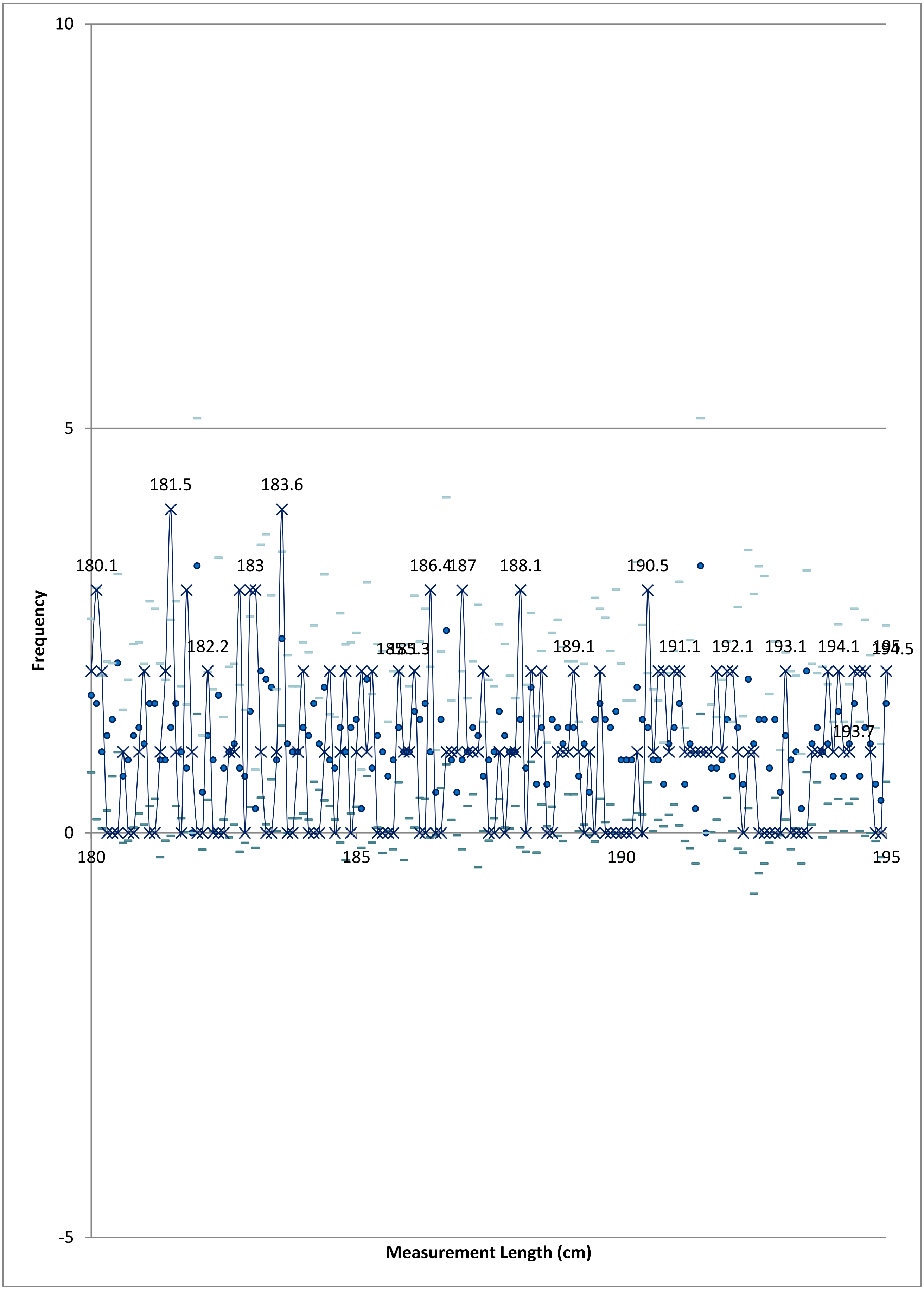


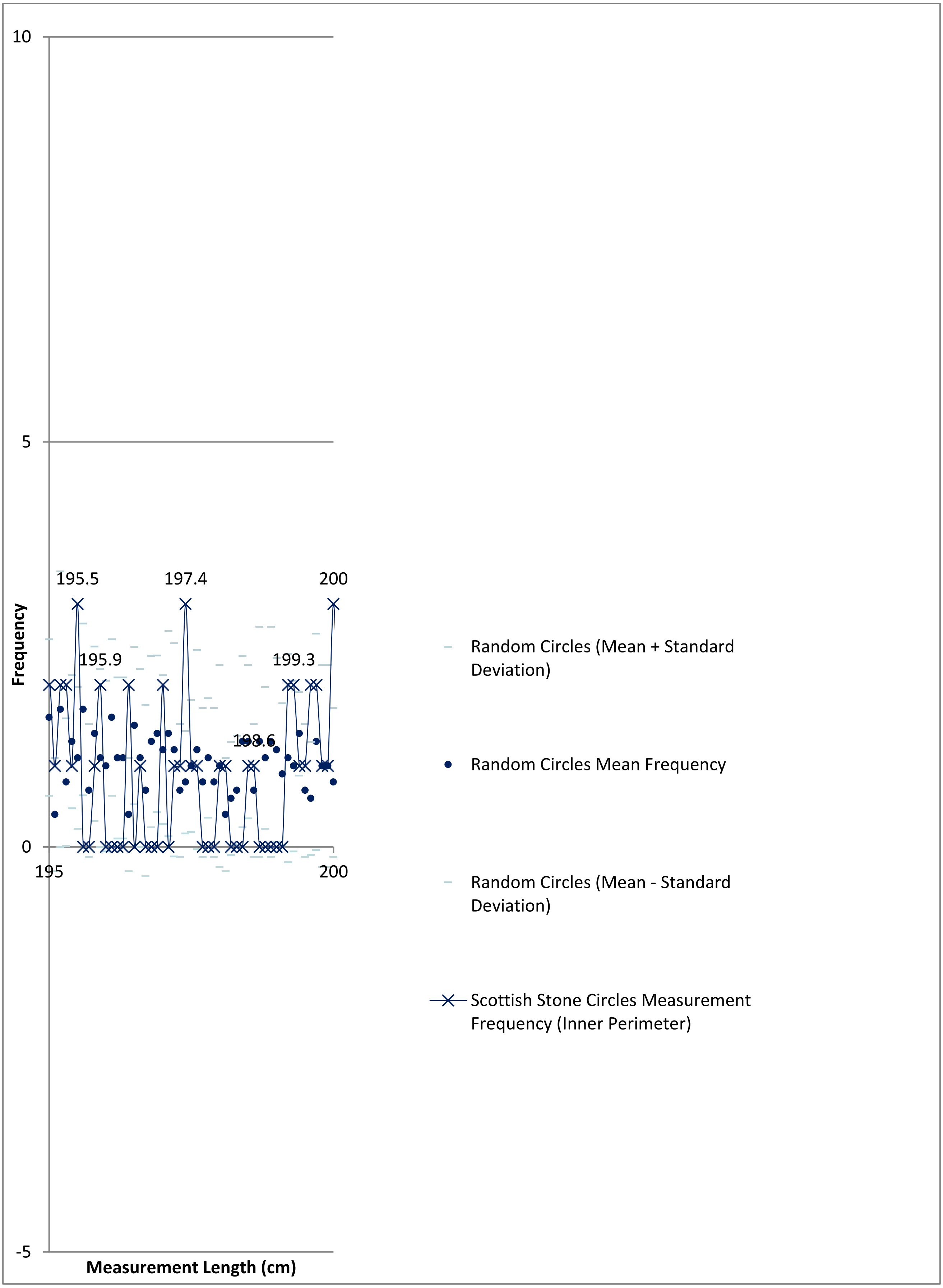
10
5
0
-5
Frequency
120
125
130
135
Measurement Length (cm)
120.1
122.4
124
123.4
125.9
127.1
129.7
130.2
133.1
133.3
121.9
126.4
128
133.5
124.7
131.4
131.9
132.8
133.8
132.5


10
5
0
-5
135
140
145
150
Frequency
Measurement Length (cm)
135.6
137
137.4
138.2
138.9
139.3
139.8
141.2
142.2
143.1
143.7
145
145.2
145.6
145.9
146.8
147.4
148
149.7
149.5


10
5
0
-5
150
155
160
165
Frequency
Measurement Length (cm)
150.8
160
153.1
157.4
159.5
164.5
150.3
151.7
152
152.5
153.5
154
154.7
156.9
160.7
161.5
162.9
164
164.2
154.5
157.2
158.8
161.3
162.1
161.7
164.9


10
5
0
-5
Frequency
165
170
175
180
Measurement Length (cm)
165.3
169.7
170.8
167.8
169.4
169.2
175.2
175
177.2
179.7
175.9
165.6
166.8
171.6
172.3
174.8


10
5
0
-5
180
185
190
195
Frequency
Measurement Length (cm)
181.5
183.6
180.1
183
186.4
187
188.1
190.5
182.2
189.1
191.1
192.1
193.1
194.1
194.5
193.7


10
5
0
-5
195
200
Frequency
Measurement Length (cm)
195.5
197.4
200
195.9
199.3
198.6
Random Circles (Mean + Standard Deviation)
Random Circles Mean Frequency
Random Circles (Mean - Standard Deviation)
Scottish Stone Circles Measurement Frequency (Inner Perimeter)