In Scotland's museums, there are hundreds of carefully carved geodesic stone balls dating from about 3000BC during the Neolithic Age. The purpose of these stone balls has not been discovered because their proposed purpose has been linked to their appearance and that has limited consideration of the ancient stone balls as being projectiles, bowls, bolas, mace heads or even ball bearings used to move megaliths. The ancient stone balls could have served some other purpose out-with our current understanding of what geodesic stone spheres might have been used for but in this case some other discovery about the balls would need to be made to indicate what that purpose could have been. When a collection of ancient stone balls is catalogued usually an image of each ball is made along with a description of how many discs have been carved into the surface, a rudimentary measurement of its diameter and a note of where it was discovered and estimated date for its carving. The measurement of the ball diameters reveals a considerable variation in their sizes with no indication of groups of balls with common diameters. Had someone been curious enough to also measure the length of the grooved pathways formed by the carved discs on the surface of the balls, it would have been found that groups of different sized stone balls had identical pathway lengths despite having different diameters. In particular many of the tetrahedral stone spheres were found to have a combined pathlength around their four-disc circumferences that gave a remarkably consistent length; a length that was known in Ancient Egypt as the Sacred Cubit (63.66cm). In carrying out this measurement, the purpose of the stone balls becomes immediately obvious as ancient rulers or winding devices; a three-dimensional ruler where a string wound around the discs results in a desired length. These rulers or winders have an appearance unlike any ruler that we might expect today which explains why measurement of length has never been considered as the possible purpose of these beautiful objects. Further measurements of pathlengths of all the other balls reveals that other common measurement lengths were commonly indicated and that by varying the number of windings around each stone ball, different integer multiples of the various measurement lengths in the Neolithic measuring system could be obtained (see Part 1). These measurement lengths are consistent with the pendulum lengths revealed from the analysis of the radii of the stone circles that were erected at the same time as the balls were believed to have been carved. The fact that such a simple oversight has turned these wonderful objects into objects of mystery shows the importance of gathering as much data as possible from even the simplest artefact and not assuming anything is unimportant.

Prehistoric Measurement of Time and Distance

The Geodesic Stone Balls of Scotland

Hundreds of carefully carved ancient stone spheres have been found throughout Scotland that are believed to date from around 3000BC. The stone spheres are of a size that fits into the palm of the hand and yet their dimensions vary significantly. Their occurrence seems to share a similar distribution pattern to that of the stone circles with the highest concentration occurring in the North-East of Scotland. Some of the stone balls are simple spheres but most are characterised by a number of carved flattened knobs or discs that are regularly arranged over the ball's surface to give forms that could be described as polyhedral spheres, most commonly in the form of tetrahedral spheres with four discs and cubic spheres with six discs. The appearance of the stone balls has been likened to spherical representations of what are commonly known as the Platonic solids, although these geodesic stone spheres pre-date Plato by around 1800 years. Some stone spheres have numerous knobs decorating their surface and there are some that have no discs at all and others which are decorated with a spiral groove running around the ball from pole to pole. The antiquity of these stone geodesic spheres and their carefully carved geometry is surprising to many archaeologists as the mathematical sophistication of these objects seems too advanced for what is generally presumed was the limited level of mathematical knowledge existing in Neolithic times. Furthermore, the purpose of these stone balls remains a complete mystery as there is nothing like them from more recent times that they can be related to apart from cannonballs, mace heads and South American bolas whose associations rely on their spherical form and the fact that the stone balls have grooves around their flattened knob surfaces that could have been used to attach a cord or leather thong.

Image 1 Range of Scottish stone balls on display at the Ashmolean Museum with ribbons showing the geometry of the carved stone

The distribution of these stone balls is concentrated in the North-East of Scotland and they are generally considered to be unique to Scotland with a few examples found in the very North of England though it appears that there is evidence for finely carved stone spheres of a similar size in many countries around the world and one example of a cubic sphere found in Bolivia (See Appendix). The distribution and their estimated age coincide with the Neolithic stone circles of Scotland which are particularly numerous in Aberdeenshire and Morayshire. The recently discovered seven knob igneous stone ball found in the ruins at Ness of Brodgar in Orkney is significant because it was found in a secure archaeological context buried with other objects beneath the North-East Buttress of Structure 10 at the Ness that can be dated from before 3000BC. Today most of the hundreds of stone ball artefacts are held in museum collections and a selection of these stone sphere artefacts displayed at the Marischal Museum of Aberdeen and the National Museum of Scotland in Edinburgh were accurately measured and are the basis of the present study.

Image 2 Camptonite Stone Ball found in situ at the Ness of Brodgar, Orkney 2013 during archaeological excavations.

A prehistoric measuring system has been proposed by the author based on the results of the mathematical analysis of the dimensions of megalithic stone circles surveyed by the Oxford professor of engineering, Alexander Thom. A series of measurement lengths related to each other through circular geometry were found to account for the radial dimensions of the stone circles of Scotland.

Pendulum Length (cm)	Circular Relationship	Period of Swing	Pendulum Type
35.96	35.96 x π = 2 x 56.48	9000 swings / 23 MD	Sidereal
41.12	41.12 x 4 = 52.36 x π	366 swings /1 MD 15000 swings / 41 MD	Sidereal
45.75	45.75 x 4 = 58.25 x π	8000 swings / 23 MD	Solar
46.35	46.35 x 2 x π = 58.25 x 5	10000 swings / 29 MD	Sidereal
50.00	50.00 x π = 52.36 x 3 50.00/Phi = 80.9 50.00 x 3 x Phi =46.35 x 2	9000 / 27 MD	Solar
52.36	52.36 x π = 41.12 x 4	12000 swings / 37 MD	Sidereal
56.48	56.48 x 9 = 80.9 x 2 x π	5000 swings / 16 MD	Sidereal
58.25	58.25 x 2 x π = 45.75 x 4	4000 swings / 13 MD	Sidereal
63.66	63.66 x π = 50.00 x 4	5000 swings / 17 MD	Sidereal
80.9	80.9 x π x 2 = 56.48 x 9	6000 swings / 23 MD	Sidereal
116.5	116.5 x π = 45.75 x 4	5000 swings / 23 MD	Sidereal
161.8	161.8 x π = 565.48 x 9	5000 swings / 27 MD 100 swings / Sun diam	Sidereal Solar

Table 1 Proposed Measurement Lengths used in Scotland in Neolithic times

These lengths, when used as pendulums, have specific periods of oscillation that give round numbers of thousands of swings for the rotation of the Earth by particular integer values of angles measured as Megalithic Degrees (where 366MD=360 degrees) relative to the stars for sidereal pendulums or relative to the Sun in the case of solar pendulums.

Many sceptics have asked, when faced with the concept that prehistoric people were numerate and used accurate measures, if measurement was important, then where are the rulers that were used by our pre-historic ancestors? As we have proposed that the measures served the dual purpose of measuring both length on the ground and time when used as pendulums, these measures could be accurately calibrated by simply adjusting the length until the required period of swing was attained for known moving stellar and solar references points in the sky. Maybe there was therefore, unlike today, no requirement for a "hard" ruler for reference. The twine or cords used for such pendulum measures would have long since decomposed over the many centuries and millennia that have passed since their use and despite the fact that pendulum bobs may have survived, they would provide no evidence of the lengths of twine that would have been attached to them and these surviving artefacts in any case may well have been classified by archaeologists as either large beads or spindle whorls rather than pendulum bobs.

Today a straight graduated linear ruler or a measuring tape would be required to obtain a cord of a length to give the required circle's radius but there are no artefacts that have survived that conform to what we might expect and recognise as one of these measuring tools. The reason for this might simply be that if it was made of wood then it would have completely decayed over a period of five thousand years and if ancient rulers were similar to those of today, we are likely to have reached the end of the road in our search for physical proof of a prehistoric measuring device. However, it is lazy thinking that, just because this is what we do today, our system represents the best if not only solution to the problem of measurement and precludes the possibility that our ancestors invented a completely different and yet workable system of measurement. A measurement consisting of a pendulum string would not survive the passing ages but perhaps some other form of ruler or winder had survived but not been recognised for what it was. This might seem like an unlikely possibility but the stone geodesic spheres have an unknown purpose and they have flattened knobs round which lengths of twine could be wound around that would potentially qualify them as some kind of measuring or winding device or given that the discs on many of these stone geodesic spheres are very shallow, that at least one measurement unit could be obtained from a single winding and then this length could be multiplied by folding the measured wound length a number of times. The stone spheres consist of almost perfectly spherical stone balls, a level of sphericity that suggests the accuracy of carving was crucially important to their purpose. The majority of the spherical stone balls have circular flattened knob-like discs cut into the surface to form tetrahedral spheres, cubic spheres and other polyhedral spheres. A multitude of possible functions have been proposed for the polyhedral spheres but the suggested uses are based solely on their spherical shapes and their true purpose remains unknown.

It is interesting that whilst archaeologists have considered that many of the stone balls are of a generally similar size, hence the suggestion of them possibly being weights, although their diameters have been measured approximately, no-one has accurately measured them or more importantly considered measuring the length of fine cord required to wind around the carefully carved flattened discs protruding from the surface of these stone spheres. The proposition that these spheres could have been used to measure lengths of twine represents a completely different concept to a linear ruler and the spherical nature of the stone balls together with variations in their diameters has probably ruled out any immediate consideration of a connection between these objects and a measuring tool. Moreover, in the absence of the knowledge of the measuring lengths used in prehistoric times, it is impossible to associate the dimensions of these objects with those unknown ancient measurement lengths though if anyone had taken the time to measure the pathlengths around the discs carved on the stone ball surfaces they would have found clusters of precisely equal lengths on spheres of different diameters that surely would have helped to suggest their purpose.

Skara Brae Artefacts

Image 3 Excavated house at Skara Brae, Orkney, previously covered by the sand dunes

The journey begins not with a sphere but with a stone artefact from Skara Brae, Orkney; the oldest surviving ruins of a village in Europe, believed to have been inhabited sometime between 3180BC-2500BC. The stone's purpose is unknown but it has a shape and pattern of decoration that resembles a tool that could have been used for winding a string around.

Figure 1 Stone artefact from Skara Brae in Orkney now at the National Museum of Scotland (X.HA 663)

A string can be wound around the tool through the perpendicularly arranged cross notches at either end of what may have served as a winding device. Perhaps the length required to complete two windings around the tool, so that the windings cross each other as they pass through the "X" shaped notches, at either end could be significant in allowing multiple pendulum measures to be obtained as well as providing a winding tool for storing a long length of twine.

The measurement of the two perpendicular windings was determined by the curator of the National Museum of Scotland from a 3D laser scan of the artefact to be 21.21cm and

21.44cm. The total length required for the two perpendicular windings is 42.65cm. Multiples of this length could be used to create multiple lengths of the proposed pendulum lengths. Table 2 shows that it is possible that the unknown stone artefact could be used to obtain multiple lengths of the different proposed pendulum measures.

Number of Windings (42.65cm)	Equivalent Pendulum lengths
3	2.01 x 63.66cm
4	3.02 x 56.48cm
7	5.97 x 50.00cm
11	13.05 x 35.96cm
11	4.03 x 116.5cm
13	11.96 x 46.35cm
15	13.98 x 45.75cm
15	10.98 x 58.25cm
16	13.03 x 52.36cm
19	10.02 x 80.9cm
19	5.01 x 161.8cm
27	28.01 x 41.12cm

Table 2

Whilst not conclusive evidence, the fact that the winding device can be used to obtain 2,3,4,5 and 6 multiples of five of the pendulum lengths and also a consecutive run of 10,11,12,13,13 and 14 multiples of a further six pendulum lengths might suggest that the dimensions of the winder were carefully chosen together with the fact that six of the pendulum lengths can be obtained through the number of windings (x) giving (x-1)or (x+1) multiples of pendulum lengths 3 windings give 2 lengths and that similarly 4 gives 3, 7 gives 6, 13 gives 12, 15 gives 14 and 27 gives 28.

There is another stone artefact (X.HA 169), found alongside the artefact (X.HA 663), shown in Figure 2 from Skara Brae Orkney, that takes the form of a spherical stone ball. Given the relationship between the dimensions of the stone artefact and the pendulum lengths perhaps a similar relationship might exist with the stone sphere where again a string could be wound around the circumference of the sphere and then known multiples of this circumference used to define known multiples off the pendulum lengths.

Figure 2 Stone Sphere (X.HA 169) from Skara Brae ex National Museum of Scotland

The diameter of the stone sphere is reported by the National Museum as 2.38", (measured using callipers and a ruler marked in eighths and sixteenths of an inch), equivalent to 2 3/8", and giving a converted circumference of 18.994cm though the accuracy of measurement is more likely to be around plus or minus 1/16th inch. The windings of a string around this stone ball can be related to the proposed pendulum lengths as shown in Table 3.

Number of Windings (18.994cm)	Equivalent Pendulum Lengths
3	**1.01** x 56.48cm
5	**2.05** x 46.35cm
8	**3.04** x 50.00cm
10	**2.98** x 63.66cm
11	**3.99** x 52.36cm
12	**4.98** x 45.75cm
13	**6.00** x 41.12cm
17	**2.00** x 161.8cm
17	**3.99** x 80.9cm
19	10.04 x 35.96cm
46	15.00 x 58.25cm

Table 3 Numbers in blue are Pythagorean triplets on which when considered as Megalithic degrees gave whole thousands of swings for the pendulum lengths involved in the ancient measuring system.

Both stone artefacts could have been used as winding measurement tools that could produce integer multiple pendulum lengths by winding a string around the objects a known number of times. The fact that 1,2,2,3,3,4,4,5 and 6 multiples of nine of the pendulum lengths can be easily obtained by less than 20 multiple circumference lengths is a useful property. The question is whether these associations occurred by design or by chance.

Other Stone Ball Artefacts

Access to two spherical stone balls, on display at the Marischal Museum in Aberdeen, was kindly provided by the museum's curator Neil Curtis.

Figure 3 Spherical Stone Sphere from Aberdeenshire ex Marischal Museum Aberdeen

This spherical ball is a simple undecorated sphere with a very fine finish giving a consistent diameter measure of 72 mm. The potential significance of this dimension is that five times the diameter of the ball represents almost exactly the 35.96cm length (within 0.4mm). A cord wound around the ball 30 times gives a length of 13 Royal Cubits within 0.5% whilst the other proposed measurement units could likewise be obtained by winding a cord around the ball as indicated in Table 4.

Pendulum Length (cm)	Number of Windings	Number of Pendulums	Measured Pendulum (cm)	Match (%)
35.96	(5x diameter)	1	36.00	100.11
41.12	20	11	41.126	100.01
45.75	93	46	45.731	99.96
46.35	41	20	46.369	100.04
50.00	20 53	9 24	50.265 49.951	100.53 99.90
52.36	30	13	52.199	99.69
56.48	5	2	56.549	100.12
58.25	18	7	58.164	99.85
63.66	14	5	63.335	99.49
80.9	25	7	80.78	99.86
116.5	36	7	116.329	99.85
161.8	50	7	161.568	99.86

Table 4

Once again as with the stone ball from Skara Brae, all twelve units can be determined to an accuracy of around 0.5% or better, however one of the measures (45.75cm) requires a very large number of windings before a multiple of pendulum lengths is obtained. Perhaps this reflects the fact that the 45.75cm pendulum measure was a solar pendulum and the ball was designed to obtain only multiple lengths of sidereal pendulum lengths. The other eleven appear to give good accuracy however the number of windings required does not form a simple easy to remember pattern. The evidence is not so convincing with this sphere as the previous balls in terms of convenience in obtaining the series of pendulum lengths with a minimum number of windings. However, the dimensions of this ball can be examined in another way.

The significance of the winding patterns should consider that certain numbers, particularly associated with Pythagorean Triplet triangle hypotenuse lengths may represent important values despite their unlikely apparent significance to our eyes. As an example, Table 4 indicates that 41 windings result in 20 x 46.35cm. It should be remembered that multiples such as 13, 17, 23, 27, 29, 34, 37 and 41 were important numbers for forming Pythagorean triplet right angled triangles and angles of arc on the circumference of stone circles using a rope with equally spaced knots for instance and therefore would have been memorable as important numbers in pre-historic times. Furthermore, it can be seen in Table 1 that these very Pythagorean triplet integers describe the angles measured in Megalithic degrees, that give whole thousands of swings for each of the pendulum lengths. If we used the 41 circumference lengths as a radius to draw a large circle, we would have a circle of 18.54m diameter (40x 46.35cm). The resulting circumference of this circle would be 58.25 metres or exactly 100 x 58.25cm and one megalithic degree would be represented by 15.916cm of arc so that 4 Megalithic Degrees could be represented by the 63.66cm pendulum measure. The particular values of Megalithic degrees such as 13, 16, 17, 23, 27, 29, 32, 34, 37 and 41 Megalithic degrees, critical to the series of pendulum lengths, can be described by the length of arc on the circles circumference where we find the following relationships:

Megalithic Degrees	Length of Arc (cm)	Number	Pendulum Length (cm)
13	206.9	4 5	52.36 41.12
16	254.7	7	35.96
23	366.1	8 7	45.75 52.36
27	429.8	12	35.96
29	461.6	10	46.35
32	509.3	11	46.35
34	541.1	15	35.96
37	588.9	5	116.5
41	652.6	4 14	161.8 46.35

Table 5

This circle size allows the important angles of arc around the circle's circumference to be accurately expressed as integer multiple lengths of the various pendulum lengths.

A second spherical ball on display at the Marischal Museum, decorated with an equilateral triangular arrangement of six small circles, was found at Insch in Aberdeenshire. Made from Greenstone, it has a diameter of 76.4 mm and a circumference of 24cm. Winding a string around the circumference three times gives 2 x 35.96cm pendulum lengths. In other words, this ball gives us a direct measure of the shortest pendulum length but requires that the string is divided in 2 by folding.

Figure 4

Like the other spherical stone ball in the Marischal Museum all the proposed pendulum lengths can be obtained from multiple windings as shown below in Table 5. Once again one special Pythagorean Triplet number, this time "27 and 17/34 on three occasions each" represent important number of windings around the circumference of the ball to produce very close to integer value multiple pendulum lengths. The associations between the dimensions of the stone balls and the proposed pendulum lengths suggests that these two stone spheres, like the stone sphere and other stone artefact from Skara Brae, could have been used as prehistoric measuring winding devices.

Pendulum Length(cm)	Number of Windings	Number of Pendulums	Actual Measurement (cm)	Match (%)
35.96	3	2	36.00	100.12
41.12	12	7	41.15	100.06
45.75	21	11	45.82	100.16
46.35	27	14	46.29	99.87
50.0	25	12	50.00	100.01
52.36	24	11	52.37	100.01
56.48	40	17	56.47	99.99
58.25	17	7	58.29	100.07
63.66	8	3	64.00	100.54
80.9	27	8	81.00	100.13
116.5	34	7	116.58	100.07
161.8	27	4	162.01	100.13

Table 6

Spherical Stone balls from the National Museum Storage Facility at Granton

The National Museum Collection Centre at Granton in Edinburgh houses over a hundred stone balls and casts of balls from other collections. The curator, Dr Hugo Anderson-Whymark, very kindly allowed the author access to the collection to measure a selection of the best-preserved stone balls. Three main types of ball were chosen for measurement namely spheres, tetrahedral spheres and the cubic spheres. Many of the stone balls have a rough and eroded finish due to the weathering they were exposed to over five millennia. In some cases, the balls are not perfectly spherical because either they were never finished or perhaps were later reutilised as grinding stones. A selection of stone balls was selected that allowed the accurate determination of both ball diameter and more importantly in the case of the tetrahedral and cubic spheres, the length of the pathway around the circumferences of the four or six discs carved into each of the ball's surfaces. Some balls which appeared initially to be spherical, were pointed out by the curator to have small flattened areas on their surface he believed may have resulted from rubbing grain. It is not known whether these stones were made for this purpose or whether perhaps they were used for grain rubbing at a later date after their original purpose had become redundant and have received wear as a result of this secondary usage. In any case these balls were omitted from the study as were any casts of balls from other collections as it is uncertain how faithfully the casting process, with its potential for shrinkage, replicates the true dimensions of the original balls. Some balls were found to be on loan to other museums whilst others, despite having good definition of carving, had damage, usually comprising a lost area due to fracturing, that prevented accurate measurement of the desired dimensions.

Seven spherical stone balls from the collection were selected for measurement. The diameter of each stone sphere was determined along six different axes using digital callipers. The length of two circumferences of each stone ball was measured using a

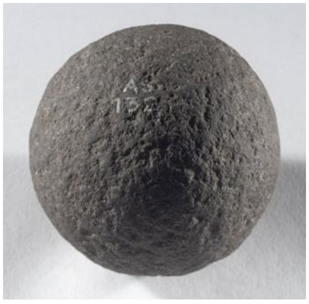
0.5mm fine non-elastic twine wound around each ball twice, each winding perpendicular to the other forming a cross at each pole on the ball's surface.

Images of six of the spherical stone balls measured

XAS 36

XAS 70

XAS 136



XAS 132

XAS 29

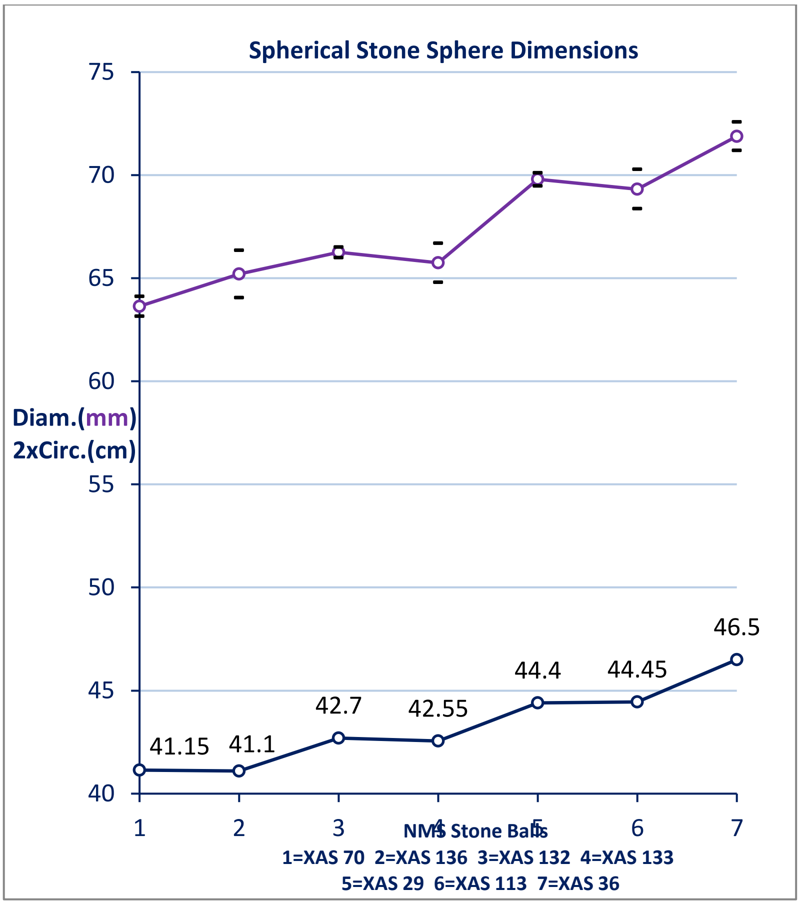
XAS 113

Figure 5

Reference	Diam (i)	Diam (ii)	Diam (iii)	Diam (iv)	Diam (v)	Diam (vi)	2x Circumference
XAS 36	7.12	7.19	7.279	7.236	7.099	7.208	46.5 46.5
XAS 70	6.356	6.377	6.428	6.374	6.280	6.370	41.1 41.2
XAS 136	6.746	6.539	6.465	6.454	6.458	6.459	41.1 41.1
XAS 132	6.628	6.658	6.594	6.599	6.645	6.629	42.7 42.7
XAS 133	6.641	6.435	6.632	6.481	6.668	6.595	42.5 42.6
XAS 29	7.021	6.944	6.960	7.014	6.983	6.955	44.4
XAS 113	6.955	7.029	7.017	6.779	6.950	6.864	44.45

Table 7

The graph shows that although there is a small but significant variation in the diameters of the measured spheres, measuring the path length, representing twice the circumference of each sphere, helps to even out variations in the ball diameter and results in three pairs of very similar lengths giving a step-like appearance to the lower curve.



Graph 1

Winding the twine around the circumference twice in a crossing pattern helps to average out any variations in spot diameters that result from hand carving the balls. The pattern of three pairs of balls with similar circumferences, despite having different diameter measurements suggests that the ball dimensions are not random and that it was the circumference of the stone spheres that was of primary importance. Looking at the lengths obtained for the circumference double lengths it can be seen that XAS 70 and XAS 136 give a length of 41.1-41.15cm equivalent to a 41.12cm pendulum length for a double winding. XAS 36 gives a length of 46.5cm for a double winding that is within 1.5mm or within 0.75mm of the desired circumference of the pendulum length (46.35cm) that may be considered to be within the experimental error of the measurement method. XAS 132 and XAS 133 give a length of 42.5-42.6cm for a double circumference winding but if instead of a double winding, a triple winding is made around these two balls a length of 63.75-63.9cm is achieved which is very close to the 63.66cm pendulum known as the Sacred Cubit. The other pair of stone spheres give a length of 44.4-44.5cm for a double circumference winding but if this is doubled, equivalent to four circumferences of the stone balls, a length of 88.8-88.9cm is obtained which gives a pendulum with a period that gives 250 swings per Megalithic Degree for solar time. Although this pendulum is not one of the standard pendulum lengths found it can be seen how useful a pendulum with this period would be in measuring any angular rotation of the Earth by simple multiplication of the desired angle measured in Megalithic Degrees by 250.

The fact that the 88.9cm pendulum length relates to solar time suggests that rather than using the stars as reference points, the Sun would need to provide the reference for measurement. Calibration could most easily be achieved by providing three vertical poles in the ground in a triangular formation with the triangle describing a known angle between two poles to the South of a third viewing pole. Given that the 88.9cm pendulum gives 250 swings per Megalithic degree it might have been convenient to lay out the three poles to mark an angle of 2 or 4 Megalithic Degrees as the 88.9cm pendulum would require 500 or 1000 swings for the time it took for the alignment of the Sun with the viewing pole and the eastern pole to be replaced with the alignment of the Sun, the viewing pole and the western pole. The way this could have been simply achieved is using a string with equally spaced knots laid out on the ground would be to make a rectangle with sides 1,29,1 and 29. Three poles can be positioned to give an angle of 2.0078 Megalithic Degrees between the viewing pole and the other two poles positioned to the South.

Table identifying the relationships between the Stone Sphere Circumferences and the pendulum lengths.

Stone Sphere	Circumference (Number, Total Length(cm))	Pendulum Length (cm)
XHA 169	3x18.9	1x56.48
Aberdeenshire, Marischal	5x22.62	2x56.48
Insch, Marischal	3x 24.0	2x35.96
XHA 663 winder skara brae	3x42.65 4x42.65	2x63.66 3x56.48
XAS 70	2x 20.55	1x41.12
XAS 136	2x 20.55	1x41.12
XAS 132	3x21.2	1x63.66
XAS 133	3x21.2	1x63.66
XAS 29	4x22.2	1x88.9
XAS 113	4x22.2	1x88.9
XAS 36	2x23.25	1x46.35

Table 8

Each of the stone spheres can be used to obtain multiple lengths of all the 12 major pendulum lengths by winding a thread around the stone sphere a given number of times, or by making one winding and then folding this length several times, thereby avoiding the problem of multiple windings crossing over previous windings resulting in increasing lengths with each winding.

The seven stone spheres measured from the National Museum Storage facility at Granton are described in the following four tables.

Pendulum Length (cm) XAS 70 and XAS 136 Circumference = 20.55cm	Number of Windings	Number of Pendulum Lengths
35.96	7	4x35.96
41.12	2	1x41.12
45.75	29	13x45.75
46.35	9	4x46.35
50.00	17	7x50.00
52.36	23	9x52.36
56.48	11	4x56.48
58.25	17	6x58.25
63.66	31	10x63.66
80.9	4	1x80.9
116.5	17	3x116.5
161.8	8	1x161.8

Table 9

Pendulum Length (cm) XAS 132 and XAS 133 Circumference = 21.2cm	Number of Windings	Number of Pendulum Lengths
35.96	17	10x35.96
41.12	33	17x41.12
45.75	13	6x45.75
46.35	11	5x46.35
50.00	33 7	14x50.0 3x50
52.36	5	2x52.36
56.48	8	3x56.48
58.25	11	4x58.25
63.66	3	1x63.66
80.9	23	6x80.9
116.5	11	2x116.5
161.8	23	3x161.8

Table 10

Pendulum Length (cm) XAS 29 and XAS 113 Circumference = 22.2cm	Number of Windings	Number of Pendulum Lengths
35.96	13	8x35.96 (8.025)
41.12	13	7x41.12 (7.018)
45.75	33	16x45.75 (16.013)
46.35	23	11x46.35 (11.016)
50.00	9	4x50.0 (3.996)
52.36	7	3x52.36 (2.968)
56.48	5	2x56.48 (1.965)
58.25	21	8x58.25 (8.003)
63.66	20	7x63.66 (6.975)
80.9	11	3x80.9 (3.018)
116.5	5	4x116.5 (4.002)
161.8	22	3x161.8 (3.019)
88.8	4	1x88.8

Table 11

Pendulum Length (cm) XAS 36 Circumference = 23.25cm	Number of Windings	Number of Pendulum Lengths
35.96	17	11x35.96 (10.991)
41.12	23	13x41.12 (13.005)
45.75	2	1x45.75 (1.016)
46.35	2	1x46.35 (1.003)
50.00	15	7x50.0 (6.975)
52.36	9	4x52.36 (3.996)
56.48	17	7x56.48 (6.998)
58.25	5	2x58.25 (1.996)
63.66	11	4x63.66 (4.017)
80.9	7	2x80.9 (2.012)
116.5	5	1x116.5 (0.998)
161.8	7	1x161.8 (1.006)

Table 12

Development of Grooved Stone Spheres

Following on from the stone spheres, perhaps the next development of the stone ball is represented by spherical stone balls that have a spiral groove carved into the surface starting at the pole of the ball and then spiralling around the ball several times before ending at the opposite pole of the ball. One stone ball with a spiral groove carved into its surface, on display at the National Museum of Scotland in Edinburgh was accurately measured. The length of the groove was 72cm which is equivalent to the length of two of the 35.96cm pendulum measures used by the stone circle builders of Scotland around 3000BC. This method of carving a spiral groove into the sphere's surface has two advantages in providing a useful measurement length, firstly the groove defines the channel where the twine is to be wound around the sphere and secondly it provides a longer path length than is available if only the circumference of the sphere or even two windings around the circumference is used.

Figure 6 Spiral decorated stone ball (AS 143) from Buchan ex National Museum of Scotland in Edinburgh

Number of Spiral Measures (72cm)	Equivalent Number of Pendulum Lengths
1	2.00 x 35.96cm
4	7.00 x 41.12cm
7	11.02 x 45.75cm
8	11.00 x 52.36cm
9	13.98 x 46.35cm
9	12.96 x 50.00cm
9	8.01 x 80.9cm
9	4.00 x 161.8cm
11	14.02 x 56.48cm
13	16.07 x 58.25cm
13	8.03 x 116.5cm
15	16.97 x 63.66cm

Table 13

Nine windings around this stone ball gives a length of twine, equivalent to integer multiples of four different pendulums. Again, a Pythagorean Triplet hypotenuse number, this time "13" is important in measuring integer multiples of two pendulum length (58.25cm and 116.5cm).

There are a few examples of stone balls carved with spirals but there are hundreds of examples of stone balls carved with four discs or six discs. Perhaps these geodesic spheres provided a development of the spiral grooved balls in providing a long pathway around discs carved into the balls surface. Rather than providing a groove in which to lay the twine the evenly spaced carved discs provide a series of four or six protruding circumferences around which a twine can be easily and tightly wound.

It is worth considering that ancient spherical stone ball artefacts may have been made for purposes other than measuring tools, such as, as grinding balls, gaming balls and more recently stone cannonballs. The difficulty in dating stone artefacts and the uncertainty associated with the use that a particular stone ball artefact was used for, makes it more difficult to find a representative selection of spheres that can be attributed to Neolithic stone balls, to test the proposal that those balls were used for measurement purposes. The geodesic spheres do not suffer the same limitations as they have a unique design, the geographic area where they were found is limited, the approximate time when they were made is approximately known by association with Neolithic sites and they are likely to have served a single purpose.

Tetrahedral Stone Spheres

The next group of stones to be examined were the tetrahedral spheres. This group of stone balls is characterised by the presence of four almost equally sized evenly spaced discs carved into the surface of a spherical stone ball. The discs are positioned in such a way as, sitting proud of the surface, to form what might be described as a tetrahedral sphere. The discs are carved into the ball's surface to provide an edge around the discs' circumferences, which varies in depth for different examples of ball but which allow a twine to be wound around in a continuous manner to give a path length equivalent to the combined length of the circumferences of the four discs. The diagram below shows the geometry of the tetrahedral sphere.

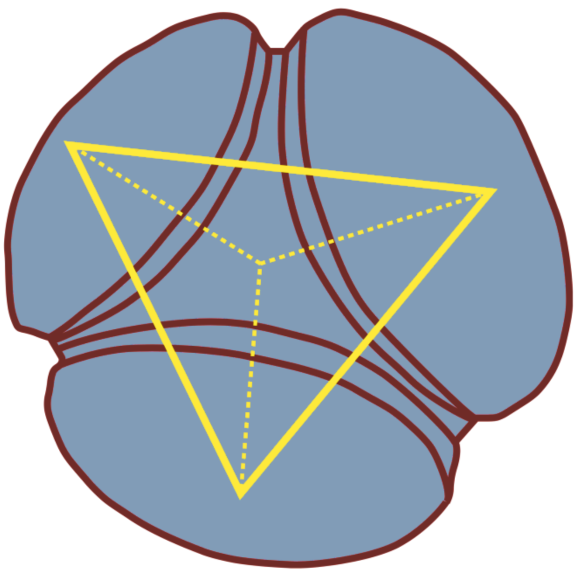
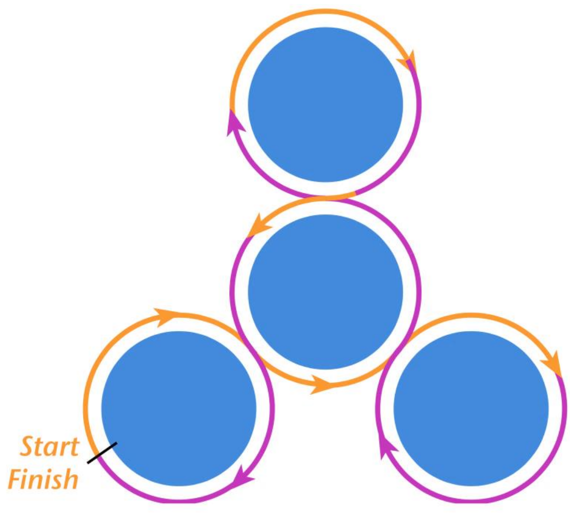


Figure 7



Windings: ½ ⅓ 1 ⅓ 1 ⅓ ½

Figure 8 The winding pattern around the four discs is represented in the following diagram showing the discs in the form of a flattened plan

Two tetrahedral spheres in the collection at Marischal museum in Aberdeen were measured. The first tetrahedral ball from Towie in Aberdeenshire made of serpentine stone is beautifully decorated.

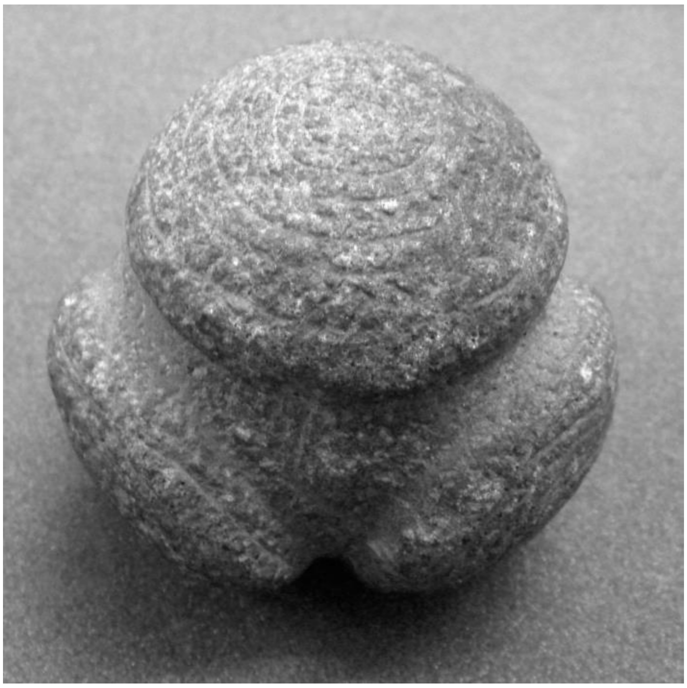


Figure 9 Towie tetrahedral ball

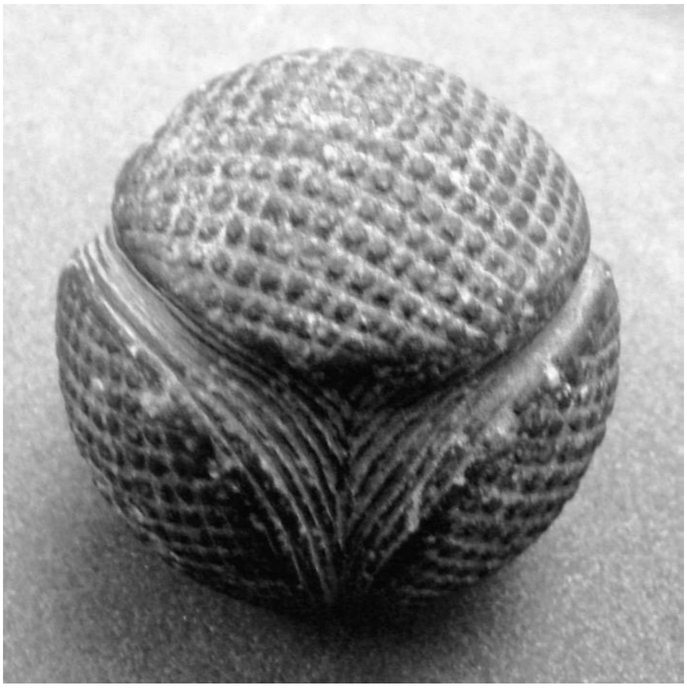


Figure 10 A second tetrahedral sphere also made from serpentine was once on display at Aboyne Castle but the location of its discovery is unknown. It is intricately carved with hundreds of circular raised dots in the manner of an ancient golf ball. A cord was wound around the four knobs in the manner illustrated above on the flattened plan of the sphere

Both tetrahedral spheres gave a path length around their four discs of 63.5-63.7cm equivalent to the Pictish pendulum length of 63.66cm.

This pendulum length obtained by winding can be accurately calibrated using the angular movement of 17 Megalithic Degrees which requires a 63.66cm pendulum to swing to and fro 5000 times. The other pendulum lengths can be obtained by dividing multiple windings in the manner outlined in Table 6.

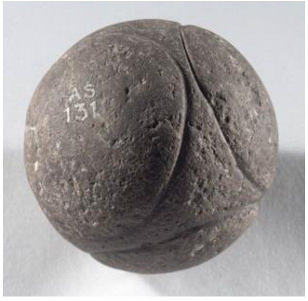
Pendulum Length (cm)	Number of Windings Around Four Discs	Number of Pendulum Lengths
35.96	17 x 63.5cm	30 x 35.96
41.12	11 x63.5cm	17 x 41.12
45.75	18 x 63.5cm	25 x 45.75
46.35	8 x 63.5cm	11 x 46.35
50.00	15 x 63.5cm	19 x 50.00
52.36	19 x 63.5cm	23 x 52.36
56.48	8 x 63.5cm	9 x 56.48
58.25	11 x 63.5cm	12 x 58.25
63.66	1 x 63.5 cm	1 x 63.66
80.9	14 x 63.5cm	11 x 80.9
116.5	22 x 63.5cm	12 x 116.5
161.8	28 x 63.5cm	11 x 161.8

Table 14

Tetrahedral Spheres from National Museum Storage Facility in Granton

Eight tetrahedral stone balls from the collection were selected for measurement

Images of tetrahedral stone balls together with their NMS catalogue reference numbers



XAS 131

XAS 71

XAS 76

XAS 153

XAS 200

XAS 171

XAS 182

XAS 61

Figure 11

The diameter of each stone sphere was determined along three perpendicular axes using digital callipers. The length of the pathway around the four incised discs carved into each stone sphere was measured using a 0.5mm fine non-elastic twine.

Reference	Diam (i)	Diam (ii)	Diam (iii)	Mean Diam.	Path Length around Four Discs (cm)
XAS 131	6.556	6.587	6.668	6.604	63.6 63.7
XAS 71	6.656	6.631	6.663	6.650	63.7 63.6
XAS 76	6.616	6.725	6.670	6.670	63.6 63.6
XAS 153	6.614	6.691	6.648	6.651	63.5 63.6
XAS 200	6.790	6.908	7.040	6.913	63.6 63.6
XAS 171	6.430	6.540	6.340	6.437	61.0 61.0
XAS 182	7.256	7.168	7.251	7.235	66.6 66.7
XAS 61	6.605	6.610	6.554	6.660	67.3 67.4

Table 15

Five of the eight tetrahedral spheres from the NMS selection have a consistent path length of 63.6cm within 1mm. In addition, the two tetrahedral spheres from the Marischal Museum collection also had a measured path length of 63.6cm meaning that 80% of the tetrahedral spheres measured had the same pendulum length (ideally 63.66cm) built into their design. This high level of consistency in measured length strongly suggests that the wound length around the four discs on this kind of tetrahedral stone sphere is the important dimension and that the diameters and circumferences of these balls has a much greater degree of variation. The diameter of the ball is therefore not absolutely pivotal in terms of defining the path length of the winding around the four carved discs because as can be seen the larger diameter tetrahedral spheres have discs that are more deeply carved into the surface, this is especially obvious in comparing XAS 200 the largest sphere carved with prominent discs and XAS 131, the smallest sphere carved with shallow discs, but both having the same path length of 63.6-63.7cm around their four discs. Another advantage in making the pathway around the discs the critical length apart from the increased pathlength possible, is that the edges of the carved discs are not subject to the same level of exposure to possible wear as the outer surface of the ball so that the pathlength around the disc edges is likely to remain constant.

Pendulum Length (cm) XAS 131,71,76,153,200 Path length = 63.66cm	Number of Windings	Number of Pendulum Lengths
35.96	13	23x35.96 (23.01)
41.12	11	17x41.12 (17.03)
45.75	23	32x45.75 (32.00)
46.35	8	11x46.35 (10.99)
50.00	11	14x50.0 (14.01)
52.36	14	17x52.36 (17.02)
56.48	8	9x56.48 (9.02)
58.25	11	12x58.25 (12.02)
63.66	1	1x63.66 (1.00)
80.9	14	11x80.9 (11.02)
116.5	11	6x116.5 (6.01)
161.8	28	11x161.8 (11.02)

Table 16

XAS 171 has a path length of 61.0cm. Although this is not one of the standard Pictish pendulum lengths, this length used as a pendulum at the latitude of Northern Scotland has a period of oscillation which gives 300 swings per Megalithic Degree with respect to sidereal time. This length would be very useful in allowing the pendulum length to be calibrated using any known angle of separation.

Additionally, the pathlength of 61.0cm can be used to obtain integer multiples of the standard pendulum lengths as shown in the table below.

Pendulum Length (cm) XAS 171 Path length = 61.0cm	Number of Windings	Number of Pendulum Lengths
35.96	10	17x35.96 (16.963)
41.12	2	3x41.12 (2.967)
45.75	3	4x45.75 (4.000)
46.35	19	25x46.35 (25.005)
50.00	9	11x50.0 (10.98)
52.36	6	7x52.36 (6.99)
56.48	12	13x56.48 (12.96)
58.25	21	22x58.25 (21.991)
63.66	24	23x63.66 (22.997)
80.9	4	3x80.9 (3.016)
116.5	21	11x116.5 (10.996)
161.8	8	3x161.8 (3.016)

Table 17

Seven of the twelve pendulum lengths can be obtained by consecutive integers of windings and pendulum lengths.

One tetrahedral sphere has a path length of 67.4cm. This pathlength can be used to obtain the standard pendulum lengths as shown in Table 10.

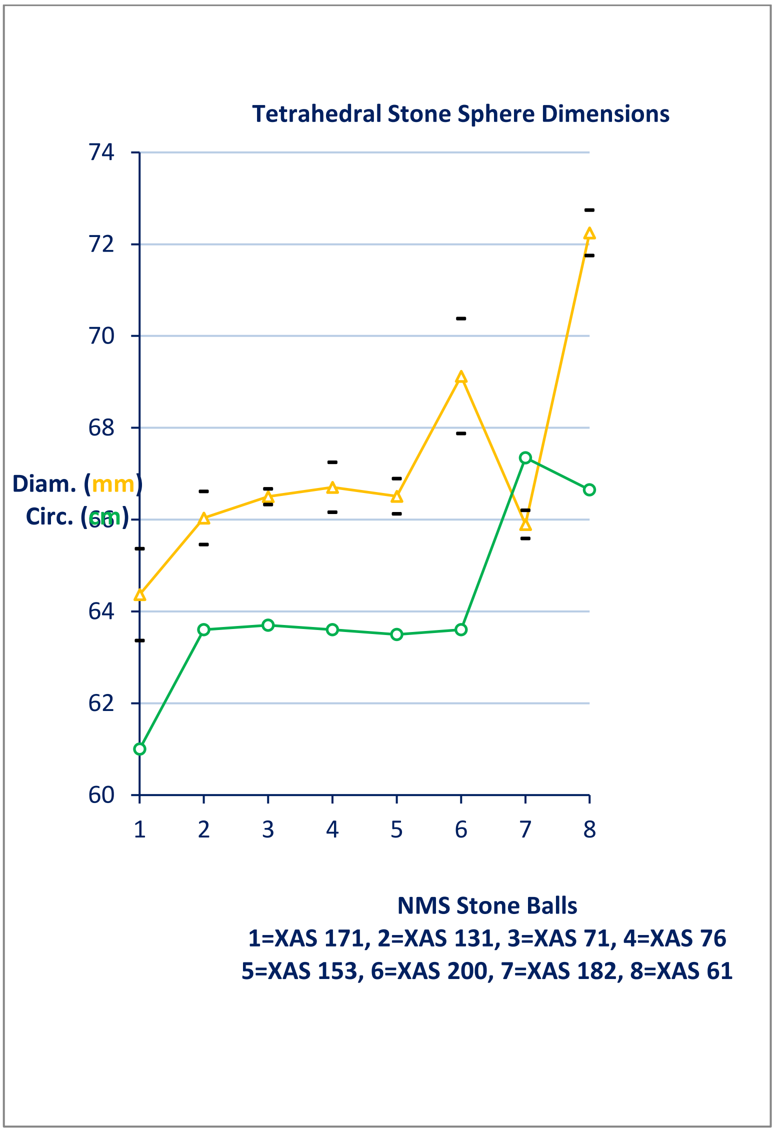
Pendulum Length (cm) XAS 61 Path Length = 67.4cm	Number of Windings	Number of Pendulum Lengths
35.96	8	15x35.96(14.99)
41.12	11	18x41.12(18.03)
45.75	17	25x45.75(25.04)
46.35	11	16x46.35(16.00)
50.0	23	31x50(31.00)
52.36	7	9x52.36(9.01)
56.48	5	6x56.48(5.97)
58.25	13	15x58.25(15.04)
63.66	17	18x63.66(18.00)
80.9	6	5x80.9(5.00)
116.5	19	11x116.5(10.99)
161.8	12	5x161.8(5.00)

Table 18

Pendulum Length (cm) XAS 182 Path Length = 66.6cm	Number of Windings	Number of Pendulum Lengths
35.96	7	13x35.96(12.98)
41.12	8	13x41.12(12.97)
45.75	11	16x45.75(16.03)
46.35	16	23x46.35(23.01)
50.0	3	4x50.0(4.00)
52.36	11	14x52.36(14.00)
56.48	11	13x56.48(12.98)
58.25	7	8x58.25(8.01)
63.66	22	23x63.66(23.04)
80.9	17	14x80.9(14.01)
116.5	7	4x116.5(4.01)
161.8	17	7x161.8(7.00)

Table 19

The tetrahedral sphere with a pathlength of 66.6cm has a large number of Pythagorean triplet hypotenuse numbers associated with the pendulum lengths. There are nine of these associated with eight of the twelve pendulum lengths. The most significant length is obtained when three pathlengths give 4x50cm lengths. In addition multiples of three of pendulum lengths can be achieved from eleven pathlengths.



Graph 2 Five of the eight tetrahedral spheres have an almost identical pathlength of 63.7cm around their four discs

The Cubic Stone Spheres

The most common geodesic spheres are those with six discs carved into the surface of a sphere forming what is commonly described as a cubic sphere. The geometry of the cubic spheres is outlined in the following diagram.

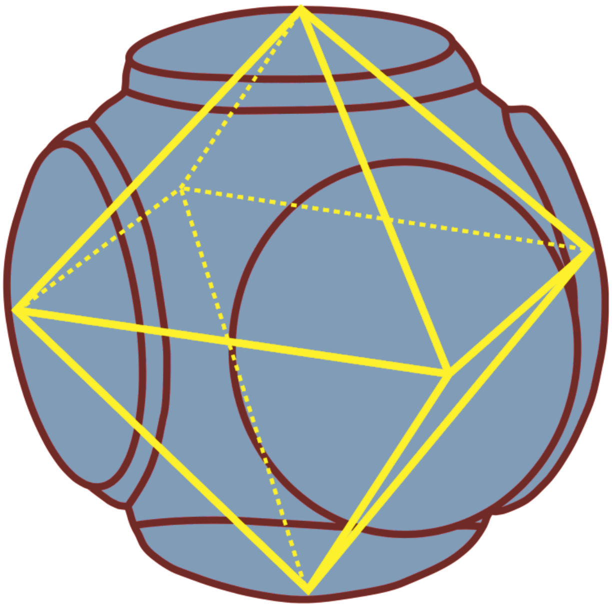


Figure 12

The winding pattern around the six discs is shown on the flattened plan of the discs below. In practice it is easier to tie a small knot at the end of the twine and place this knot at "A" then make an anticlockwise winding of the first disc and cross over the knot and continue the winding pattern shown. In this way the twine is held firmly in position on the ball and allows the twine to be more easily wound around the remaining five discs. the knot also defines the starting point of the path length.

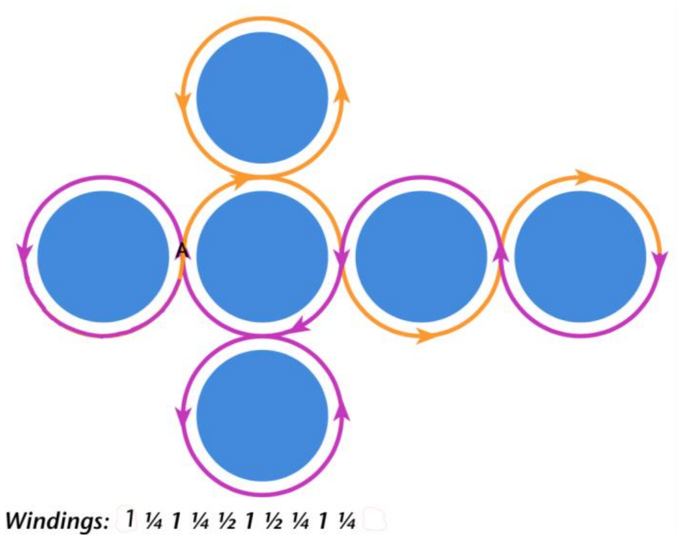


Figure `13 Pattern of winding used to determine combined length of six carved discs

Cubic Sphere Stone Rulers

Three cubic stone spheres on display in the Marischal Museum, Aberdeen were carefully measured. Formed from granite, the exact origin of the find is unknown but is reported as originating in the North East of Scotland.

Figure 14 Cubic Sphere stone ball, North East of Scotland

A length of non-elastic cord was wound around each of the six knobs and measured. The way in which this is achieved is shown in Figure 7. The pattern of winding is critical as the path has to go around each knob's circumference without retracing its path. Each circular knob was separately measured to see whether each knob had identical circumferences. The length required to go around all six knobs was 83.2cm. If this winding is repeated three times in total, a length of 2.496 metres is achieved or 5 metres (50.0cm x 10) with 6 windings. Three of the knobs had circumferences of 14.0cm, 13.7cm and 13.4 cm, respectively winding around these three knobs gives a pendulum length of 41.1cm. Seventeen windings of the six knobs are equivalent to 27 Royal Cubits the significance of this being that both 17 and 27 are Pythagorean triplet hypotenuse numbers. It is also significant that 17 windings are also equivalent to 25 x 56.48cm pendulum lengths.

Number of Windings around the 6 discs (83.2cm)	Number of Pendulum Lengths
1	2.02 x 41.12cm
3	4.99 x 50.00cm
5	8.98 x 46.35cm
7	10.00 x 58.25cm
7	5.00 x 116.5cm
11	20.00 x 45.75cm
13	16.99 x 63.66cm
16	37.01 x 35.96cm
17	27.01 x 52.36cm
17	25.04 x 56.48cm
35	36.00 x 80.9cm
35	18.00 x 161.8cm

Table 20

The Pythagorean Triplet hypotenuse numbers 13, 16, 17, 27 and 37 are important to the relationships between four of the pendulum lengths and the number of windings required to obtain integer values of those pendulum lengths.

A second cubic sphere in the Marischal Museum of unusual design is made of sandstone and reported as originating from Cairnroben, Banchory Devenick. The length around the six knobs was 83.4cm or within 2mm of the previous cubic sphere measured. Again, as with the other ball, 3 windings gave almost 2.5 metres (5x50cm). Once again, the circumference of the knobs is different and the three smallest knobs have circumferences of 13.1cm, 14cm and 14 cm again giving a total of 41.1cm, equivalent to a 41.12cm Pendulum length.

Figure 15 Cairnroben Stone Ball (Cubic Sphere)

A third ball from Fyvie , another cubic sphere, with six discs was measured though two of the knobs are damaged. This time the length of winding a string around all six knobs was substantially different from the other two cubic spheres measured being 88.9cm.

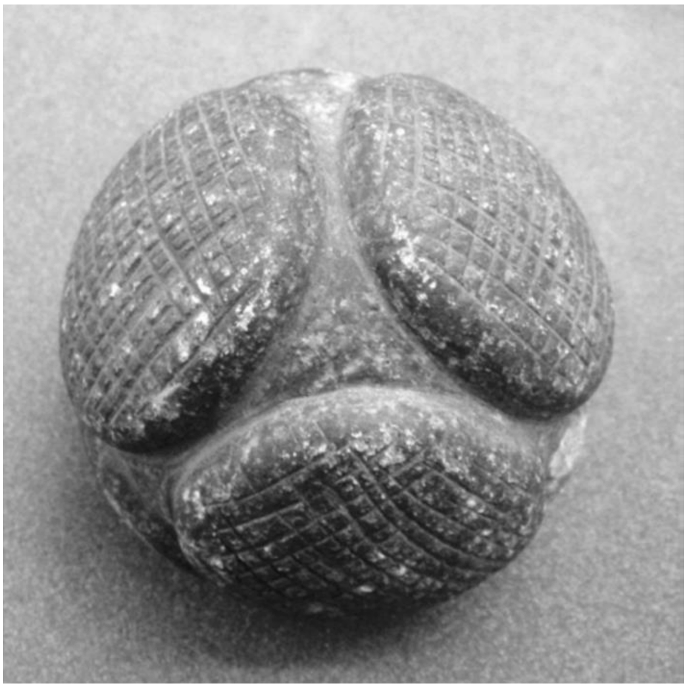


Figure 16 Cubic Sphere from Fyvie

The following table details again how multiples of the different pendulum lengths can be obtained as a function of the number of windings.

Number of Windings around 6 Discs (88.9cm)	Number of Pendulum Lengths
5	6.98 x 63.66cm
6	12.97 x 41.12cm
7	11.02 x 56.48cm
9	16.00 x 50.00cm
9	10.99 x 80.9cm
10	16.98 x 52.36cm
12	23.02 x 46.35cm
17	42.03 x 35.96cm
17	33.03 x 45.75cm
17	12.97 x 116.5cm
19	29.00 x 58.25cm
20	10.99 x 161.8cm

Table 21

The Pythagorean Triplet Hypotenuse numbers 13(twice), 16, 17(four times), 23 and 29 appear to be intimately involved in the relationship between the number of windings and the number of pendulum lengths produced by those windings. It seems remarkable how the numbers 13, 16, 17, 23, 27, 29, 34, 37 and 41 are involved in the number of windings required around the discs and the number of pendulum lengths obtained by the windings.

Nine windings around the six knobs is equivalent to 8 metres. Six windings are approximately equal to 13 x 41.1cm pendulum lengths (41.03cm) When only five of the knobs are used instead of six, five windings (25 knobs) is equivalent to 7 Royal Cubits.
The ball could be used to measure all the proposed pendulum lengths, furthermore when the individual discs were measured it was found that three of the four intact knobs had a circumference of 16cm whilst the fourth is smaller having a circumference of 15.5cm. By winding a cord around the three larger knobs once and the smaller knob once a length of 63.5cm or close to the 'Sacred Cubit' length of 63.66cm is obtained.
Whilst some cubic spherical balls are made along similar lines, others are designed slightly differently and by clever choice of dimensions have a greater versatility allowing them to be used as more complicated calculators. Perhaps there is a development in this measuring technology with passing time or the complexity may simply reflect the proficiency of individuals living in separate small communities, each using the same general technology, but expressing their mastery over the "science" of measurement through their own distinctive carved spheres, in the same manner as the later "master-pieces" of the masons. It seems unlikely that these measurements occur by chance over such a large geographical area and we conclude that it is these measurements that have been purposefully designed into the stone balls in order for them to be used as long rulers. The form of these unique rulers as stone polyhedral balls represents the perfect design for formers that can be used to wind long measuring lines to obtain multiple lengths of the twelve different pendulum measures.

Pitts River Museum Cubic Sphere

A black basalt stone ball from the North East of Scotland and at one time displayed in the Pitt Rivers Museum in Dorset and now in a private collection was accurately measured. The density of the stone ball was measured at 2.96g/cm3 which is consistent with igneous basaltic rock. The length of the path around the six discs was determined to be 87.6cm using a 1mm plastic coated copper wire wound carefully around the discs.

The stone ball first exhibited at the Pitts River Museum, Dorset can be described as a cubic sphere, with six indented discs arranged on the surface of a 7.0cm diameter basalt sphere.

The 1mm gauge wire was wound around the ball following the path of the six indented circular discs in the manner previously prescribed.

Figure 17 Basalt Cubic Sphere the North East Scotland ex Pitt Rivers Museum Dorset

The length of fine copper wire required to complete one circuit around the six discs was 876mm. This length is interesting because it can be used as multiple lengths to create integer multiples of the proposed pendulum lengths which then could be used to draw circles on the ground to make circular structures such as stone circles, passage cairns or circular huts. The dimensions of this stone ball are clever in that certain integer multiples are common to four of the pendulum lengths and the other multiple integers are small and easily remembered.

Pendulum Lengths (cm) Pitt Rivers ball Path Length = 87.6cm	Number of Path Lengths	Number of Pendulum Lengths
35.96	7	17.052x35.96
41.12	8	17.043x41.12
45.75	12	22.977x45.75
46.35	9	17.01x46.35
50.00	4	7.008x50.00
52.36	3	5.019x52.36
56.48	11	17.061x56.48
58.25	2	3.008x58.25
63.66	8	11.008x63.66
80.9	12	12.994x80.9
116.5	4	3.008x116.5
161.8	13	7.038x161.8

Table 22

Here another Pythagorean Triplet number (17) is important but this time in describing 17 lengths of four separate pendulums by 7, 8, 9 and 11 windings around the six discs.

Another special number, 13 describes thirteen pendulum lengths of 80.9cm and 161.8cm from 12 and 14 windings around the six discs. A third special number, 23, is involved in the design of this cubic stone sphere as 12 windings give 23 lengths of the 45.75cm pendulum. Most importantly for this stone sphere two windings are equivalent to three pendulum lengths of 58.25cm; the pendulum length when used as a radius gives a circumference of 366cm, so that the circumference can be marked out as megalithic degrees each megalithic degree represented by 1cm.

Cubic stone Spheres from the National Museum of Scotland collection facility in Granton


XAS 155


XAS 203


XAS 114


XAS 115


XAS 192


XAS 194


XAS 107


XAS 141

Figure 18

Eight Cubic spheres in the National Museum Store collection at Granton were kindly made available to the author by the curator for careful measurement.

Reference	Diam (i)	Diam (ii)	Diam (iii)	Pathlength (cm)
XAS 155	7.126	7.177	7.051	84.5 84.2
XAS 203	6.751	6.691	6.690	87.6
XAS 114	7.220	7.027	7.165	88.9
XAS 115	-	-	-	88.9
XAS 192	6.717	6.676	6.762	88.8
XAS 194	7.200	7.100	7.177	88.9
XAS 107	7.276	7.198	7.284	88.0
XAS 141	7.288	7.266	7.275	88.1

Table 22

XAS 114, 115, 192 and 194 like the cubic sphere from Fyvie in the Marischal Museum all have a pathlength around the six discs of 88.8-88.9cm. The significance of this length is that when used as a pendulum length it has a period of 300 swings/Megalithic Degree relative to the Sun, that is relative to solar time. There are two cubic spheres XAS 107 and XAS 141 that have a path length of 88.0-88.1cm. These two pendulum lengths also have a period that gives 300 swings/Megalithic Degree but this time the pendulum is used at night time measuring sidereal time measuring the movement of the Earth relative to the stars. It is interesting that again despite the variation in diameters of these cubic spheres, the path lengths within the two groups is constant. The pendulum lengths obtained from winding a twine around the six discs of these two groups of balls could be further calibrated using a triangular arrangement of vertical rods positioned so that a viewing rod at the northern vertex of the triangle can align with two rods at the southern vertices of a triangle and the angle separating the two rods from the viewing rods is an integer number of Megalithic Degrees and then either the movement of the Sun aligned with the viewing rod and the eastern most rod to the South until the later solar alignment with the viewing rod and the western-most southern rod can be followed by counting the number of swings of the 88.9 cm pendulum, when using the alignment of the Sun or counting the number of swings of the 88.1cm pendulum relative to the apparent movement of a star.

Pendulum Lengths (cm) XAS 107 and XAS 141 Path Length = 88.1cm	Number of Path Lengths	Number of Pendulum Lengths
35.96	9	22.049x35.96
41.12	7	14.998x41.12
45.75	13	25.034x45.75
46.35	10	19.008x46.35
50.00	4	7.048x50.00
52.36	3	5.048x52.36
56.48	9	14.039x56.48
58.25	2	3.025x58.25
63.66	13	17.991x63.66
80.9	11	11.979x80.9
116.5	4	3.025x116.5
161.8	11	5.990x161.8

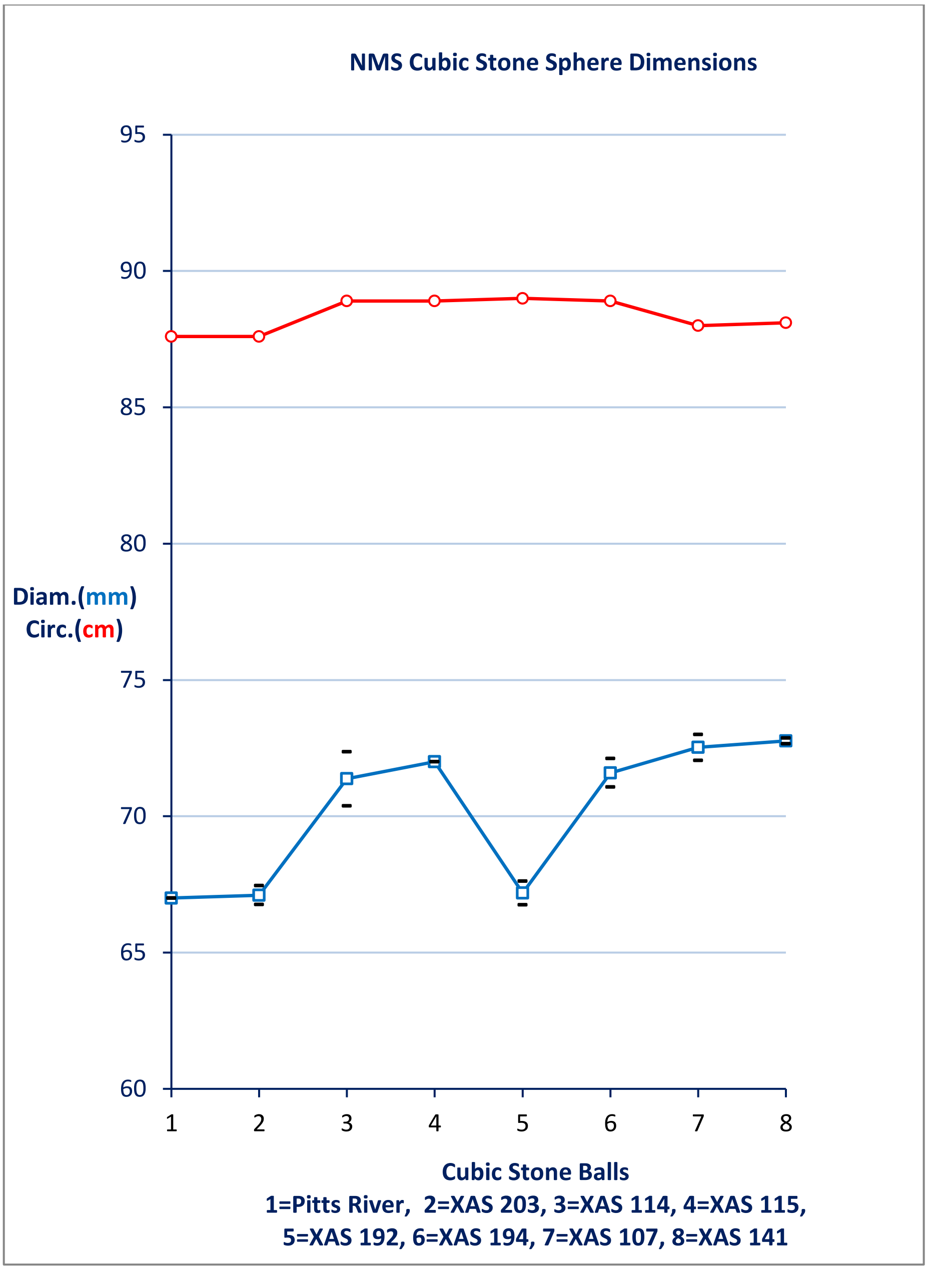
Table 23

XAS 203 has a pathlength of 86.7cm which is the same as that of the Pit Rivers ball previously described in Table 13.

XAS 155 has a path length of 84.3cm. The significance of the length is not immediately obvious but Table 17 shows that when the relationship between this length and the standard pendulums is examined two windings around the six discs gives a length equivalent to 3 x 56.48cm and three windings gives a length of 4 x 63.66cm.

Pendulum Lengths (cm) XAS 155 Path Length = 84.3cm	Number of Path Lengths	Number of Pendulum Lengths
35.96	3	7.033x35.96
41.12	20	41.002x41.12
45.75	13	23.954x45.75
46.35	11	20.006x46.35
50.00	16	26.976x50.00
52.36	18	28.98x52.36
56.48	2	2.985x56.48
58.25	9	13.025x58.25
63.66	3	3.973x63.66
80.9	24	25.009x80.9
116.5	18	13.025x116.5
161.8	25	13.025x161.8

Table 24



Graph 3

The way in which the stone balls must have been used was to obtain one pathway length of the six discs and then to take this length and fold it over the required number of times to obtain an integer multiple of the desired pendulum length. It would not be possible to have large multiple windings around the six discs on most stone balls because the carved discs are too shallow. On stone balls where the discs are more prominently carved it may have been possible to make multiple windings of the same pathway but then there is the likelihood that the windings will become progressively longer with each wound pathway as the twine will tend to overlap previous windings at certain points along the incised pathway leading to inaccuracy in the measurement lengths obtained. So perhaps all these geodesic spheres required one careful winding which was then multiplied by folding to obtain multiple lengths of the different pendulum lengths.

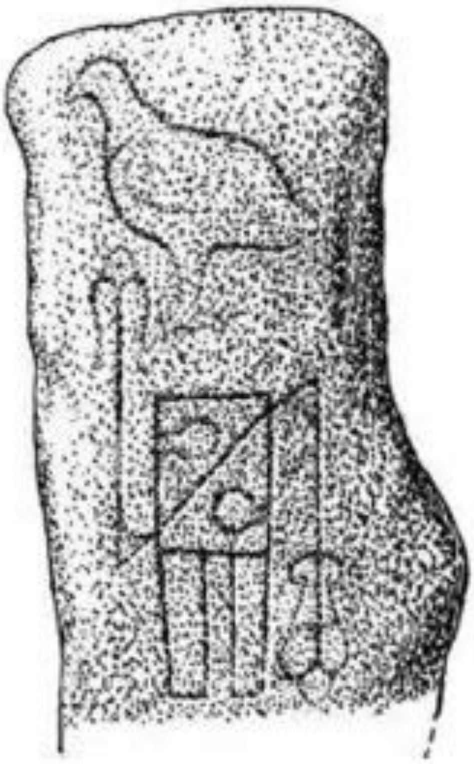
Part II

The Bronze-Age Development of Pictish Winding Devices

Class I Pictish Stones

Evidence for the proposed measuring system carved on the Class I Pictish stones. The Z-Rod symbol carved with the Snake, Double Disc and Notched Rectangle symbols is believed to represent a compound pendulum carved with the three constellations of Draco, Cetus and Vela. These three constellations contain star pairs separated by Hour Angles that could have been used to calibrate the Pictish pendulums. The distance between the hooks and the apple shaped bob on the Z-Rod coincide with the pendulum lengths that constituted the measuring system. There is however no evidence of any carved symbol representing geodesic spheres on the Class II stones. Given that the measuring system based on pendulum lengths seems to have survived from 3000BC to the time of the Class II Pictish symbol stones, it might be expected that unless the same carved geodesic stone spheres were still in use or that new ones continued to be carved, that a new winding device may have been developed by this time and that if this was the case then that winding device may have been represented by one of the Class I Pictish symbols.

The Notched Rectangle

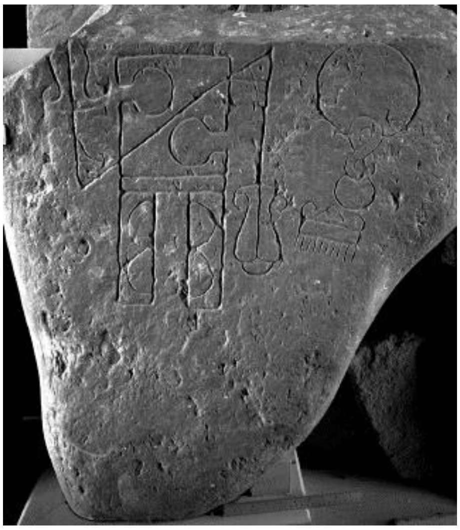


Birnie Stone

Migvie Stone

Newton House

Westfield



Clynemilton

Ballintomb Stone

The Scottish geodesic stone balls proposed as winder measuring devices dating from 3200-3000BC may have been used for many centuries or longer, but what happened after they became redundant with respect to long measurement? Are there artefacts that resemble the stone ball measuring devices or were the measuring spheres simply replaced by straight rulers?

Going forward in time to around 1200BC there is the physical evidence provided by the Pictish Class I symbol stones but there is nothing carved on these megaliths that resembles the stone geodesic spheres. Perhaps the old geodesic stone spheres were still in use but there is also the possibility that a new development of these hard-to-make spheres had been developed. One symbol, known as the Notched Rectangle, has an unusual appearance containing two round circles in the upper half of the symbol that touch the sides of the rectangle and a wide notch that bisects the bottom half of the symbol. Three stones from Clynemilton, Westfield and Birnie show a consistent pattern of circles in communication with the sides, a fourth stone, the Migvie Stone has two half circles cut out of either side of the rectangle. The diameters of the two carved circles are interesting in that they are different and measure 8.3cm and 7.0cm. It is possible that if the Notched Rectangle is representative of a wooden board with the circles and notch cut out of that board that it could represent a tool used for the easy manufacture of two clay balls. There are short channels between the edges of the rectangle and the proposed circular cut-outs. These channels would allow a stick covered in a ball of clay or a mixture of clay and straw to be rotated and for a sphere to be formed in the circular cut-out. The two balls formed by each of the two circular cut outs could then be dried and a final thin layer of clay put around the ball, to overcome the shrinkage occurring during drying, and reworked in the circular cut-outs to obtain a pair of finished balls of the desired diameters. The stick could then be cut off and the two balls could then be positioned in the notch so that they touched each other and a twine wound around the balls in a figure-of-eight pattern to obtain a length, in a similar manner to that obtained by winding a twine twice around the stone spheres or once around the discs carved into the surface of the tetrahedral and cubic spheres. There are no surviving artefacts of either the proposed wooden ball former or the clay balls that may have been made in Scotland and the hypothesised function therefore is no more than interesting conjecture. Searching for ancient clay ball artefact led to the discovery of two terracotta balls from Israel that were hollow and whose surfaces showed evidence of having been turned. The balls had diameters perpendicular to the turned axis of 8.3cm and 8.85cm giving circumferences of 26.1cm and 27.8cm. It is interesting that again one ball has a diameter of 8.3cm and the significance of this dimension is that a twine wound around the circumference twice gives a length of 52.2cm or very close to a Royal Cubit (52.36cm). The Notched Rectangle symbol was identified in a previous paper as the symbol representing the constellation of Vela and interestingly there is a deep space object in this constellation known as NGC 3132, the Eight-Burst nebula, which may be linked to the idea of placing two balls together and winding a twine around them in a figure-of-eight pattern in the event (see later paper) that this deep space object was visible in pre-historic times.

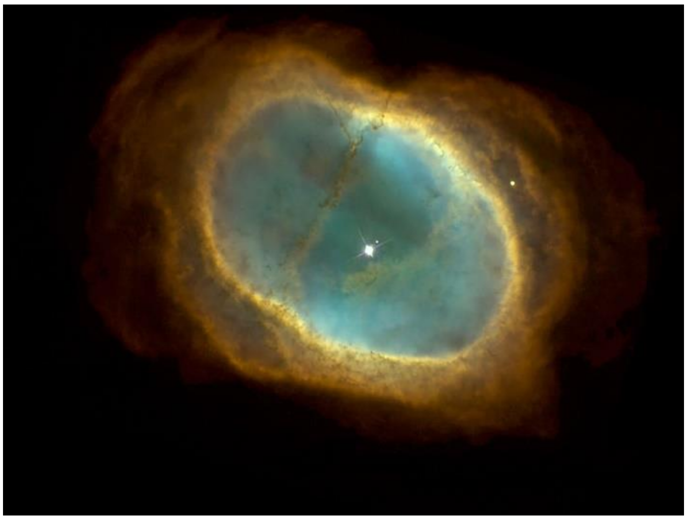


Image 4 Eight Burst Nebula NGC3132 in Vela

A fine twine wound around the two terracotta balls, from the Israeli desert, in a figure-of-eight pattern around their circumferences gives a length of 53.9cm. This measure can be used to obtain the pendulum lengths used in Scotland from around 3000BC.

Number of Windings of 53.9cm	Number of Pendulum Lengths
2	35.96cm x 3 (2.998)
13	41.12cm x 17 (17.040)
17	45.75cm x 20 (20.023)
6	46.35cm x 7 (6.977)
13	50.00cm x 14 (14.014)
34	52.36cm x 35 (35.000)
22	56.48cm x 21 (20.995)
13	58.25cm x 12 (12.029)
13	63.66cm x 11 (11.007)
3	80.9cm x 2 (1.999)
13	116.5cm x 6 (6.015)
3	161.8cm x 1 (0.999)

Table 24

Figure 19 Clay ball maker with terracotta balls in the positions they would have been revolved around when wet on sticks.

Terracotta balls sitting in the carved hollows which allows a twine to wound around the two balls in a figure of eight pattern.

Class II Pictish Stone Evidence of Winding Device

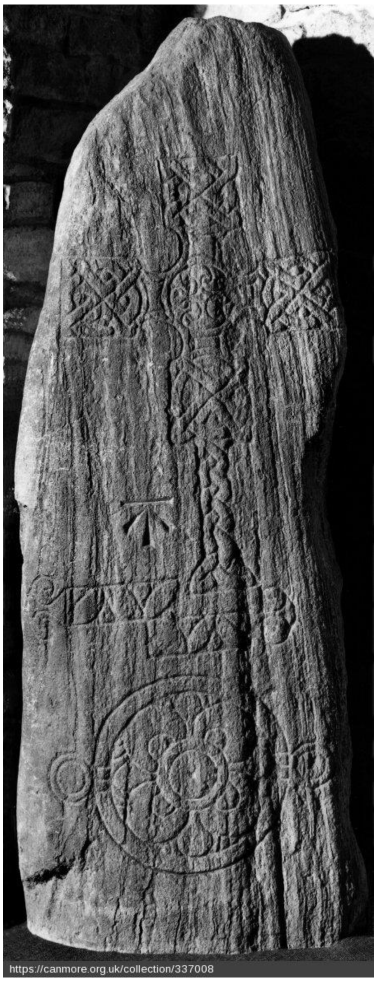


Figure 20 Class II Pictish Stone in Bourtie Kirk dating from around 850-950AD showing what may be a winding/measuring device fitted with petal shapes that resemble the curved paths carved on the geodesic stone balls dating from around 3000BC

The Monmusk symbol stone has an unusual, stepped, crank shaped symbol that appears to have a handle at each terminal. The stepped shaft appears to be decorated with petal shapes, formed from quadrant circular arcs, that resemble the curved paths carved on the geodesic stone ball some 3800 years earlier. No artefact resembling the carved symbol survives but given the diameter of the carved circle defining the petal shapes is 16.7cm the arc length described by one edge of the petal is one quarter of a Royal Cubit. This measurement combined with the two handles, does suggest that the symbol could represent a long-forgotten winding device. The six petals have the equivalent total arc length of three Royal Cubits. However, it is likely that given the crank like configuration of the device, complete with two handles on the terminals, that the device is likely to have been designed to rotate as the twine was unwound from the petals. In which case the same pattern of petals is likely to have existed on the other side of the crank as well. The way in which a string could have been wound around the petals is shown in the diagram below.

It is only by constructing the device that one can see how it might have been most easily operated. Although there are many ways in which twine can be wound around the petals, in order to keep the winder rotating about its longitudinal axis, the pathway of twine needs to alternate round petals on either side of the device in a way that allows the twine to easily unwind from the device through the gaps between the petals.

The easiest pathway found involves winding a twine around the outer side of the petal, then crossing over to the other side but continuing to wind the twine in an arc around the next petal to form a semi-circular path. Each side of the petal represents one half turn of the winder and four petal lengths represent one Royal Cubit in length. The way the prototype winder was constructed involved supplying it with four tubes containing a metal ball that passed through the winder. Each time the winder made a half revolution the four balls fell through the tube and made a sound as they hit the bottom of the tube. In this way it was easy to count the number of revolutions of the winder by simply counting the number of clacking sounds made by the balls.

It is interesting to find that if one starts winding from an origin at the centre of the winder the way in which the device revolves as the string is unwound has a period of six 180 degree half-rotations away from the operator followed by six 180 degree half-rotations towards the operator followed by a further six half-rotations away and then a final cycle of six half-rotations towards the operator to complete the unwinding of one layer of twine around the length of the device. Having completed four cycles of six half rotations, each of the twelve petal edges will have had a length of string unwound and this is equivalent to six Royal cubits in length. Further windings can be put onto the winder, assuming that the petals sit proud of the device and that there is sufficient depth of channels to take more than one winding of string, in the same manner, and longer lengths obtained with multiples of four half turns representing one Royal cubit. The operator is required to rotate the winder in alternating cycles of clockwise and anti-clockwise directions, changing the direction of rotation every six revolutions. Perhaps the easiest way to count the number of lengths of Royal Cubit measures, given that six half revolutions gave 1.5 Royal Cubits, was to count two series of six counter revolutions and to measure out multiples of 3 Royal Cubit lengths. The person unwinding the string could simply count the number of half revolutions he made but there is another carved feature on the Monymusk stone that may have made the counting easier. There are four small circles carved on the crank shape which may have represented closed end tubes that could have contained a small heavy ball. In this case, following each half rotation, the ball contained within each tube would fall to the bottom of the tube and make a noise as it struck the end. The balls would be free to move along the tube hitting either end of the tube as the cranking device continued to rotate, making a sound for each half-revolution which could more easily be counted. The choice of materials of construction for the tubes and balls would control the sound produced making a ringing noise perhaps for metal or a rattling sound for wood or bone. This interpretation however is merely more supposition based on intuition about how such a device might have been made and we cannot know for certain whether the carved pattern on the Monymusk stone does in fact even represent an ancient winding ruler or not.

The carved stone images of the Monymusk stone on the Canmore site were examined. A rubbing of the stone complete with scale was used to determine the dimensions of the stepped would give a circumference of the circle, or four petal arcs of one Royal Cubit. A model of the proposed winding device was cut from oak. A 1cm thick wooden board, marked with six squares with 8.33cm sides, was cut in the stepped shape seen. There is no way of knowing the thickness of the petals carved on the stone but petals were drawn out on a 4cm thick piece of oak and cut out and sanded to give a precise arc length on each petal edge of 13.1cm. Twelve petals were assembled on the step shaped board, six on either side in the same arrangement as carved on the stone. A small gap of 2mm was left between the petals and this was achieved by removing a small amount of wood and rounding off the tips of the petal. Two handles were carved from oak and fitted to either end of the device. Finally, four oak tubes were made with an internal diameter of 14mm. A metal ball of 12mm was placed inside each of the four tubes and each tube was sealed with a disc of cow horn.

The device made is a very unusual looking object and is quite heavy but it does serve as a winding device when a hemp line is wound around the petals, in a prescribed pattern, as the line is pulled from the winder, it rotates in cycles of six half-turns in one direction and then in the opposite direction for the next six half-turns. As the twine is unwound every 12 rotations 3 Royal Cubits are unwound. The counting of rotations is aided by the sound of the four metal balls making a clunking noise as they fall through the wooden tubes and hit the horn ends with each rotation.

Figure 21 Proposed winding device made in oak using the symbol dimensions of the stepped symbol carved on the Monymusk Class II Pictish stone.

Part III
The Pi-Calculator Comb

It has been shown how pendulum lengths used as integer multiple lengths could account for the radial dimensions of the Neolithic stone circles of Scotland. It has also been shown how these multiple lengths could be obtained by winding a twine around stone geodesic spheres. Whilst the measurement lengths used formed a series of lengths related through circular geometry, so that one measurement unit used as a radius automatically produced a circle whose length of circumference was an integer multiple of one of the other measurement units, there may have been occasions when a particular length of circumference was required and the radius that produced a circle of that circumference was needed in order to draw that circular circumference on the ground. Today of course it is known that the relationship between a radius of a circle and its circumference is given by

$$C = 2\pi r$$

Where **C** is the circumference

r is the radius and

π is approximately 3.1427

Today the radius that gives a desired circumference can be calculated by division and then measured using a ruler onto a length of string or a tape measure and then scribed into the earth to form a circle. The question is whether in pre-historic times man had the ability to make such arithmetic calculations and if not, how he might have achieved such a mathematical procedure using some other practical method. The circle was of central importance to Neolithic people inhabiting Scotland some five thousand years ago and it has been shown how the hundreds of stone circles were constructed as calendrical devices and how megaliths placed on the circumference were used for alignment purposes with the Sun on the horizon. Many circles had circumference lengths that could be divided into arc lengths that were equivalent to integer multiples of one of the measurement lengths whose length in turn corresponded to an integer multiple of Megalithic Degrees of arc angle. This allowed the movement of the Sun and the stars rising and setting from and into the horizon to be conveniently measured.

There is an interesting class of artefact that has been found that has the appearance of an everyday object whose purpose has never been questioned. The comb. Whilst most comb artefacts found did serve the primary purpose of combing hair either human or sometimes wool, there are some combs that appear to be different in certain respects. The first thing is they have a relatively large number of teeth, which might be accounted for if they were used as nit combs, the second common feature is that these combs have in common is the pattern of decoration that adorns their spines which consists of arrangements of dots and circles that form triangular and square shapes in line with the patterns of stars in the Summer and Winter Triangles and the Square of Pegasus. The third and most important feature of the combs is that they have precisely the same number of teeth, namely forty-three. This number of teeth could of course be coincidental but it is interesting to consider why such an apparently random number of teeth could be useful if the comb served another purpose.

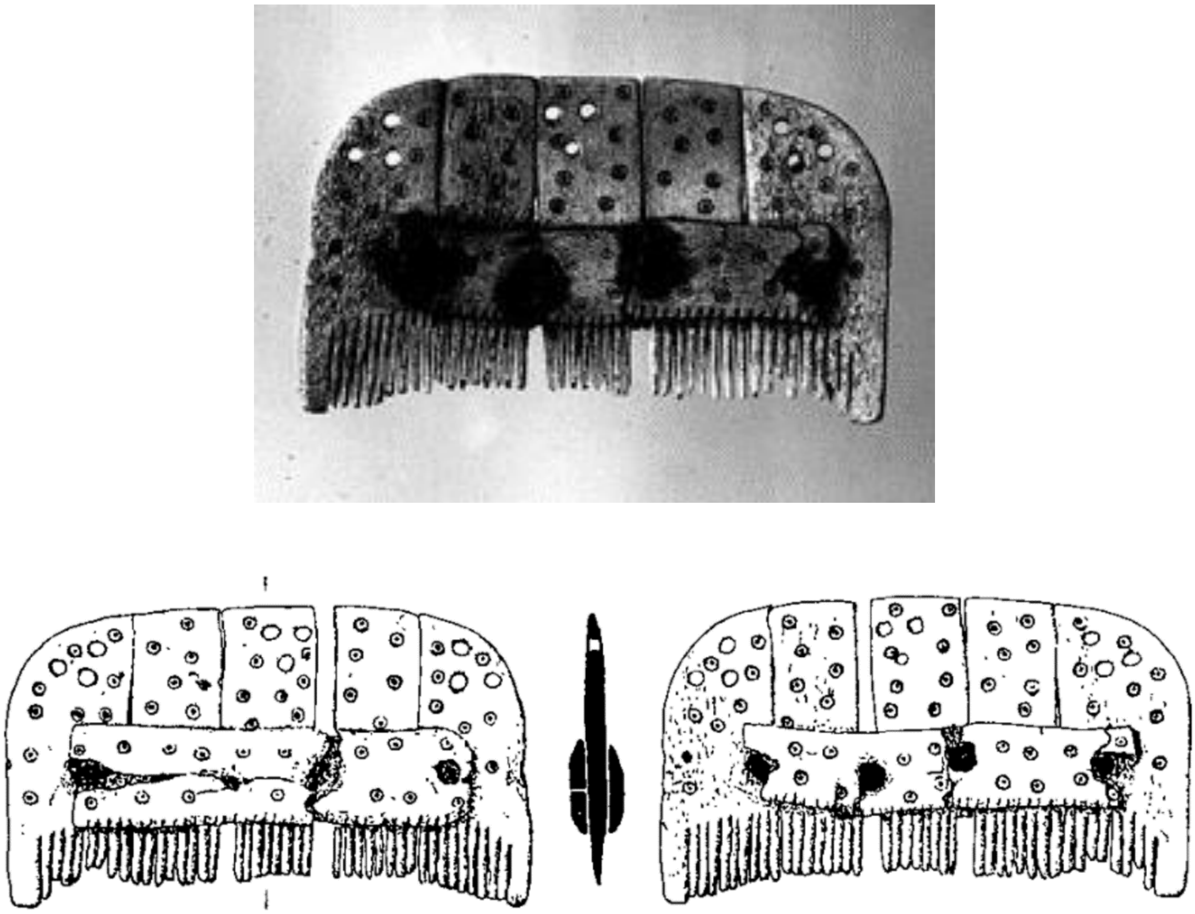


Figure 22 Comb recovered from Burrian Broch, North Ronaldsay Scotland showing pattern of circles and dots and three sets of holes arranged as triangles. National Museum of Scotland

There is a particularly interesting comb made from bone that was found at Burrian Broch in North Ronaldsay in Orkney. The first remarkable thing about this comb is the pattern of decoration on the comb with cup and circle-type carved images all over the comb, a symbol that we are confident represents stars, much in the style of the ancient stone megaliths decorated with similar cup and ring marks. There are 44 of these circular patterns over the main body of the comb on one side and sixteen along the comb's spine that keeps the five pieces of bone that comprise the combs needles together. There are three triangular arrangements of three holes that pierce through sections 1, 3 and 5, that is, the two end pieces and the central section. This appears to represent the Summer

Triangle moving across the night sky through the seasons. The Summer triangle consists of the three bright stars Deneb in Cygnus (the Goose), Altair in Aquila (the Eagle) and Vega in Lyra (the Horseshoe) so their position on the comb seems to replicate their position in the sky remarkably well for these three main festival dates and this appearance is heightened by the elliptical curve shape of the "comb" handle.

The three stars comprising the Summer Triangle are precisely aligned with cardinal positions on the following days:

Festival	Date 1200BC	Star	Alignment
Mid Winter	Dec 21st	Vega	Due East Dawn
Spring Equinox	Apr 1st	Altair	Due South Dawn
Mid Summer	Jun 21st	Deneb	Due South Dawn
End of Summer	Aug 6th	Deneb	Due West Dawn
Autumn Equinox	Oct 3rd	Deneb	Due South Dusk

Table 25

The pattern of three stars on the comb can therefore be interpreted as representing the position of the Summer Triangle at dawn as the seasons move from the mid-winter festival when it is due East (as portrayed on the left-hand position on the comb) to the Spring Equinox when it is due South at the central position on the comb, to the End of Summer Festival when the Summer Triangle is due West (depicted on the right-hand side of the comb) and then back to its central position on the comb due South at the Autumn Equinox but now at dusk rather than dawn.

Whilst the decoration is beautiful, its astronomical portrayal of the passing of the year may suggest that the comb could have had a more serious use than as a means of combing hair. The question is how a comb might be used as a mathematical tool.

If a person had a long length of a fine twine consisting of a multiple number of pendulum lengths that he wished to form a circumference of a circle, and he wanted to know what radius length was required to make a circle of that circumference, he could wind the length of twine through the teeth of the 43-tooth comb in such a way as to form twenty-one long loops between alternate teeth and have the ends of the length of the twine hang down as a two single strands at the first and last gap of the comb. The 21 loop ends could be threaded onto a straight bone needle and gently pulled down to form a common length of loop, whilst at the same time the single strands of twine hanging down at the end of the comb rise up shortening as the loop lengths increase, until the strands and loops all form the same length. Once the two ends of twine and the twenty-one loops are the same length, the circumference has in effect been divided into 22 equal parts consisting of 21 loops and 2 half loops consisting of the two end strands. It is now a simple procedure to find the radius of the circle that results in a circumference of the desired length of twine, by taking the seven middle loops of the comb and using that length as a radius to draw a circle. The reason for this is that the comb is a good tool to allow the very near approximation of pi (22/7) to be achieved.

$$\pi = 3.141593.$$

$$22/7 = 3.142857 \; (1.0004x\pi)$$

There are a couple of other good examples of combs that could have been used as pi-calculators in converting circumference lengths to radii. One again was recovered from the ruins of a broch at Buckquoy and the third example is a Coptic wooden comb from Egypt dating from the 6th-7th Century AD.

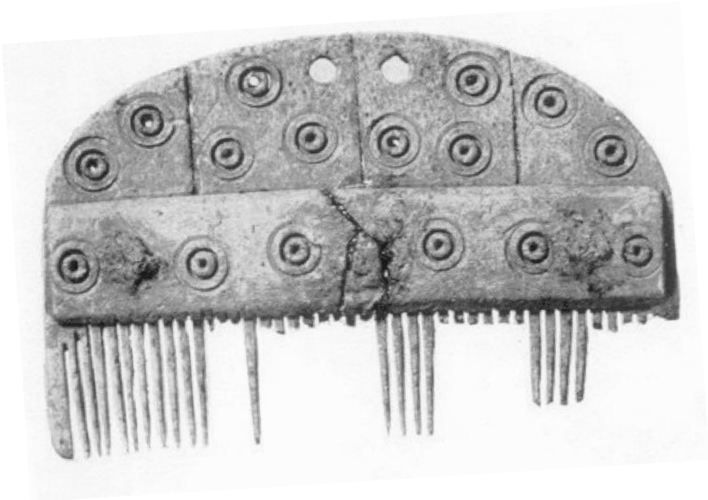
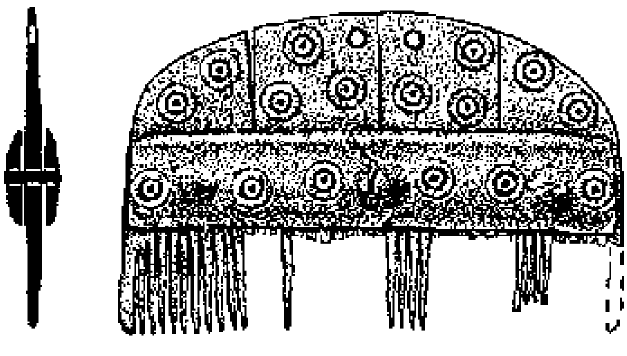


Image 5 of the Buckquoy comb from the paper by Anna Richie describing the archaeological excavation at the Buckquoy Pictish-Norse farmstead in 1976. The comb shows evidence that it originally had 41 teeth, 43 teeth including the outer teeth.



Drawing of Buckquoy Comb showing its construction from four pieces of bone cut with teeth sandwiched and pinned between two horizontal lengths of bone

The Buckquoy comb was recovered from a Pictish and Viking-age farmstead in Buckquoy, Orkney. This distinctive type of comb has been found on three other sites in Orkney: the Broch of Burrian, North Ronaldsay (as already illustrated)

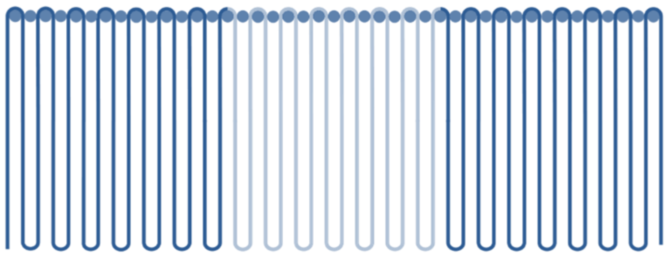
This distinctive type of comb has been found on three other sites in Orkney: the Broch of Burrian, N Ronaldsay, the Broch of Berwick (Anderson 1883, fig 213) and the Brough of Birsay. It would appear to have been a local Orcadian fashion, most probably among the Pictish population, and does not appear to have been found outside Orkney (thus militating against MacGregor's suggestion (1974, 80) that the form may have been derived from one-piece combs found in S Scotland). The other comb-fragments from this phase belong to the more common double-sided composite type, again of native origin; nos 55 and 56 show linear incised decoration

Image 6 Comb found at Dun Cuier, Barra, Outer Hebrides 600-800AD National Museum of Scotland

The Dun Cuier comb is a beautifully decorated example of a single edge arched comb decorated with 21 dotted circles and a further ten dotted circles on its connecting bar. There are intricately cut patterns in the comb but most importantly there appear to once more have been 43 teeth in the original comb.

Method of Winding Twine between the Comb Teeth

The 43 teeth could be used to form 21 loops and 2 single strands at either end in the way proposed so that 7 loops give the radius length corresponding to the circumference length of twine wound between the comb teeth. However the other side of the combe which has only 11 teeth could also be used to form 10 loops with 2 single strands and three loops and one single strand taken from either end of the comb is equivalent to the radius of the circle as the length corresponds to 7 single strands whilst the 10 loops and 2 strands

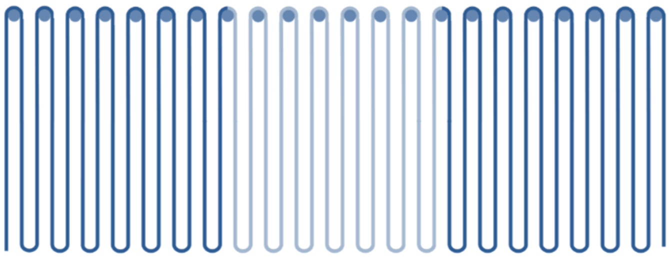


Pattern of Winding loops of twine through a 43 tooth comb to produce the Radius of the circle that gives rise to this length of circumference.

There is another common style of Pictish comb that consists of a double edged comb constructed from sections of bone sandwiched and pinned together between two horizontal lengths of bone. This double-edged style of Pictish comb is the one commonly represented as the Comb Symbol on Pictish Stones dating to 1200BC.

This comb has 22 teeth along one edge and appears to have had 43 teeth along the other edge

Image 7 Double sided bone comb from Freswick Links, Caithness dated to 800-1100AD. National Museum of Scotland



Twenty-two tooth comb showing winding of twine to produce 21 loops and 2 half-loops. The seven loops shown in the centre of the comb are equivalent to the radius of the circle that has a circumference equivalent to the length of 22 loops.

It is evident that any comb with a sufficient number of teeth could be used to create the required number of loops, that is a comb with more than 11 teeth but in this case the comb would not lend itself so elegantly to the required division and the person using the comb would have to know to form either ten loops plus two half loops or 21 loops and two half loops.

As an example, the comb shown below, which is a Pictish comb found at Buiston Crannog, a round house made on man-made islands in lochs, is decorated with the usual pattern of triangular dots and circles but has 35 teeth on the bottom edge which seems to go against our theory about such combs being used as pi calculators, but it can be seen in this example how the central 22 teeth appear to be conveniently separated from the teeth on either side by intentionally carved larger gaps

Image 8 Bone double-edged Comb from Buiston Crannog, Ayrshire 600-800AD

National Museum of Scotland. Note the 22 central teeth on the bottom edge

The fact that there are combs with 22, 31 and 43 teeth which are quite often decorated with patterns of the “summer triangle” of dotted circles suggests that there were some combs that were designed with the main purpose of being useful tools in determining the lengths of radii of circles that had a desired circumference.

Evidence of Combs Carved on Class I Pictish Symbol Stones

There are very few combs that have survived the passing millennia, and fewer still that have survived in a sufficiently intact condition to enable the determination of the number of teeth originally cut. There are however combs that have been carved as symbols on the Class I Pictish stones and they can be examined to see whether there is any indication of their use as potential pi-calculators. Previous analysis of the Class I Pictish stones has revealed the date they were carved as being around 1200BC.

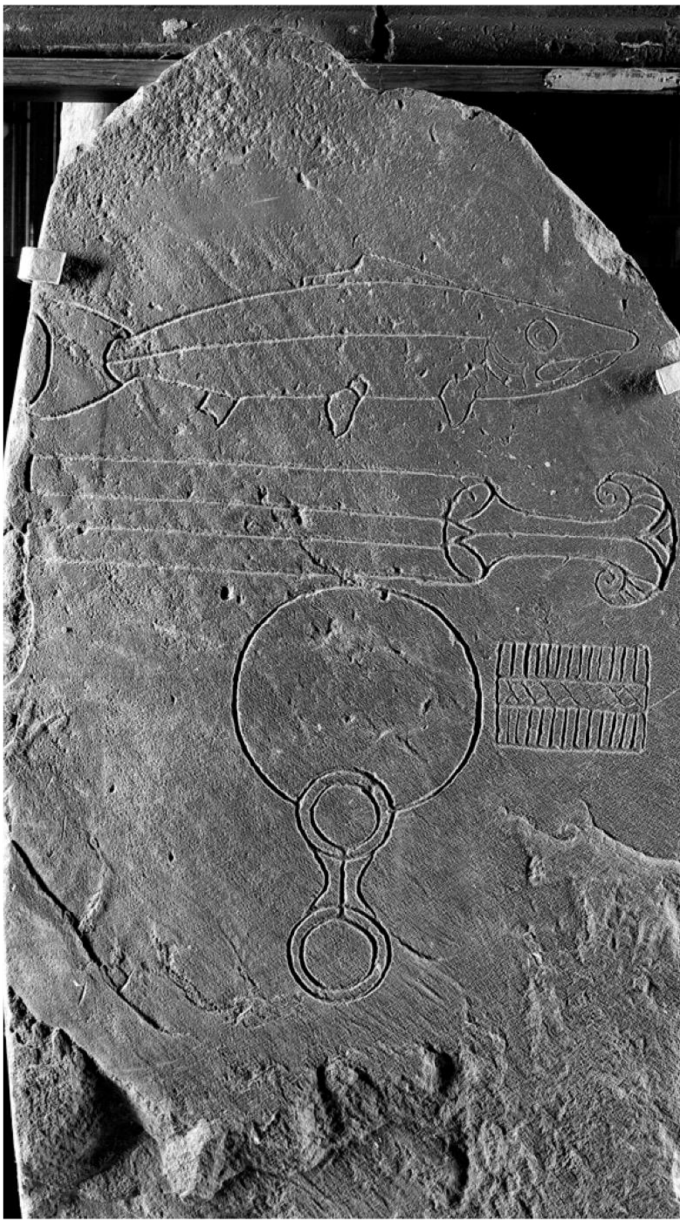
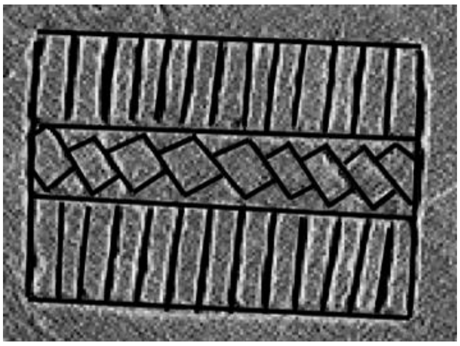


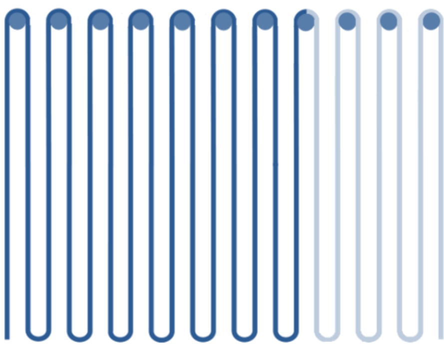
Image 9 Class I Pictis stone from Dunrobin

It appears that the level of detail employed in carving teeth on a Pictish comb symbol is usually quite basic. The Pictish Comb symbol is relatively small in comparison to other symbols, reflecting the relatively small size of the constellation of Coma Berenices compared with, for instance the constellation of Virgo depicted by the Pictish Mirror symbol which lies directly beneath Coma Berenices. It is impossible to carve a realistic representation of comb teeth on a granite stone when the carved lines themselves are about 5mm thick and it is not unreasonable that a general, readily identifiable representation of a comb's shape was made. Indeed, it seems like the detail included in the Pictish Comb's design was more often directed towards portraying the constellation of Coma Berenices either through the representation of the comb's spine as a right angle replicating the pattern of stars in the constellation or more commonly the inclusion of a pattern of nine lozenges decorating the comb replicating the occurrence of a cluster of nine stars known as the Nine Maidens or MEL 111.



Detail of the Dunrobin Pictish Comb showing the 9 Lozenges and 11 teeth on both edges

There is a good example of a carved comb on the Class I Pictish Stone from Dunrobin. The symbol of the comb carved on the Dunrobin stone is a particularly good example of its association with the constellation of Coma Berenices with Nine lozenges representing the nine MEL 111 cluster in Coma Berenices. Furthermore, eleven teeth have been carved on either side of the comb, or rather twelve gaps between the teeth which allow ten loops and two single end strands to be formed that can be used to divide a length of twine equivalent to a desired circumference to be divided into 22 equal parts. Seven of those parts made up as a single end strand plus three loops can be used as a radius to draw a circle that gives a circumference equivalent to the length of twine wound through the comb teeth.



Pattern of Winding a Twine of the Circumference length around Eleven Comb teeth to obtain the Radius that gives a Circle of the desired circumference.

The eleven teeth winding gives 10 loops and 2 single end strands of the same length. The end three loops and single strand give the required radius length or alternatively seven loops can be taken and folded in two to obtain the radius.

The idea that in Scotland over a thousand years and possibly over two thousand years ago the inhabitants of Scotland used a comb to divide circle circumference lengths into 22 parts which could then be used as a radius by taking seven of those parts to draw a circle with the desired circumference is at first a strange idea to our modern way of thinking. The possibility that an object we take completely for granted as a comb for hair could possibly serve a mathematical function seems ridiculous until we look at the way in which the number of teeth cut on these combs and the way in which loops can be formed between the teeth so perfectly serves the purpose of replicating the division of a length by pi to obtain a radius.

The question arises as to whether similar combs were made in other areas of the world.

Egyptian Coptic Comb dated between 600-800AD

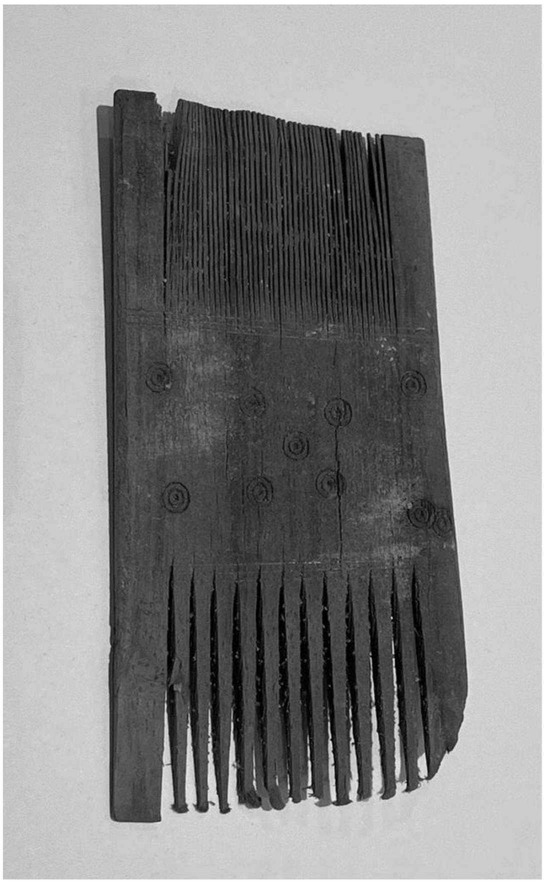


Image 10

This comb, has eleven teeth on one side and forty-three three teeth on the other side allowing both suggested methods of dividing a circumference into 22 parts and then selecting 7 of those parts to form a radius length. Perhaps the different comb edges allow different thicknesses of twine to be used with the 43 teeth allowing only very fine twine to be used whilst the 11 teeth have much larger gaps between them allowing twine of thicknesses of around 3mm to be wound between the teeth. In many respects the comb is similar to the double-edged bone combs of Scotland even to the extent of the decoration of the comb with dots and circles, in this case there are nine dots within nine double circles with the central five dots forming a square with a central dotted circle. The Egyptian comb is taller than it is wide and is formed from a single piece of cedar wood, a material that would not have survived the ravages of time in Scotland.

Ritchie, A 1976, Excavation of Pictish and Viking-age farmsteads at Buckquoy, Orkney, Proc Soc Antiq Scot, 108 (1976-7) 174-227

MacGregor, A 1974 The Broch of Burrian, North Ronaldsay, Orkney, Proc Soc Antiq Scot 105 (1972-4) 63-118

Part IV
A Roman Measuring Device

The Roman Dodecahedron

Figure 23 Bronze Roman Dodecahedron found in Jublains, France

The Scottish geodesic stone balls proposed as measuring, winding devices dating from around 3200-3000 BC may have been used for many centuries if not millennia and later perhaps they were replaced by clay spheres that were easy to make and continued the tradition of winding twine around spheres to obtain pendulum lengths but what happened after they became redundant with respect to long measurement? Is there any other artefact that resembles the stone ball measuring device or were the measuring spheres simply replaced by straight rulers? Perhaps as the Stone age became the Bronze age there are bronze artefacts that were made that shared a similar purpose that has not been recognised for what they are simply because the purpose of the stone ball rulers themselves were unknown until the recent discovery outlined in this paper. There is such a group of bronze objects that date to around 200-300AD that are associated with the Roman empire but whose purpose remains unknown. Around a hundred bronze dodecahedrons with small knobs located at each of the twenty vertices have been found throughout central Europe and the British Isles. These objects share important characteristics with the geodesic stone spheres although they have more complex geometric form being predominantly dodecahedrons and an icosahedron rather than the cubic and tetrahedral forms of the earlier stone spheres The bronze Roman dodecahedrons have small knobs at each of their twenty vertices that are ideally suited to winding a cord around in a similar fashion to the way in which a cord was The bronze Roman dodecahedrons have small knobs at each of their twenty vertices that are ideally suited to winding a cord around in a similar fashion to the way in which a cord was wound around the flattened knobs.

Pes	(Roman) foot	1 pes	296 mm
palmipes	foot and a palm	1 ¼ pedes	370 mm
cubitus	cubit	1 ½ pedes	444 mm
gradus pes sestertius	step	2 ½ pedes	0.74 m
passus	pace	5 pedes	1.48 m
decempeda pertica	perch	10 pedes	2.96 m
actus (*length*)		120 pedes	35.5 m
stadium	stade	625 pedes	185 m
mille passus mille passuum	(Roman) mile	5000 pedes	1.48 km
leuga	(Gallic) league	7500 pedes	2.22 km

Table 26 shows the measurement units used by the Romans. It can be seen that all the measurements relate to one basic measure namely the “pes” or Roman foot but for longer measures the cubitus, the gradus and passus are likely to have been used. We might expect that a winding measuring device might allow the basic unit of the Roman foot to be identified and/or the Roman cubit and Roman pace which would allow long distances such as the stade (185m) and Roman mile (1480m) to be measured using multiples of the cubit (x410) and the passus (x1000).

Around a hundred bronze dodecahedrons ranging from 4-11cm in size have been found throughout Europe mostly in Germany and France and a few as far North as the Scottish border. One icosahedron has also been found but all examples share a common feature in that they originally had a small bronze knob located at each of the vertices. The knobs are ideally suited to winding a thread around in a similar manner to the way that a thread is proposed to have been wound around the carved channels on the geodesic stone spheres. The question is what pattern or patterns of winding would produce lengths that are equivalent to known Roman measurement lengths.

With a dodecahedron there are twenty vertices, and a pathway can be found called a Hamiltonian cycle or circuit which visits each vertex only once. The pattern can be represented in two dimensions in the diagram below. The thick blue lines represent the circuit and the thin blue lines the edges of the dodecahedron. The black circles are the knobs at each vertex and they are numbered to show the pathway to be followed.

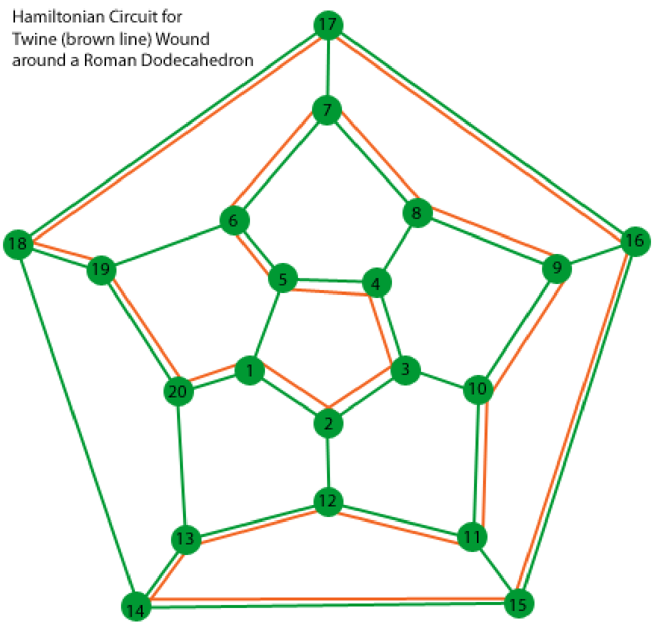


Figure 23 Hamiltonian Circuit with knobs numbered to show the order of winding

Figure 24 Roman Dodecahedron replica wound with fine thread in the pattern of a Hamiltonian circuit required a length of 59.2cm or 2x29.6cm or 2 Roman feet to complete the circuit.

There are other possible pathways or circuits around the dodecahedron knobs that can be followed using a thread. One interesting circuit is the formation of five pointed stars or pentagrams using the five knobs of each pentagonal face. Five pentagrams can be formed on five adjacent pentagonal faces as shown below.

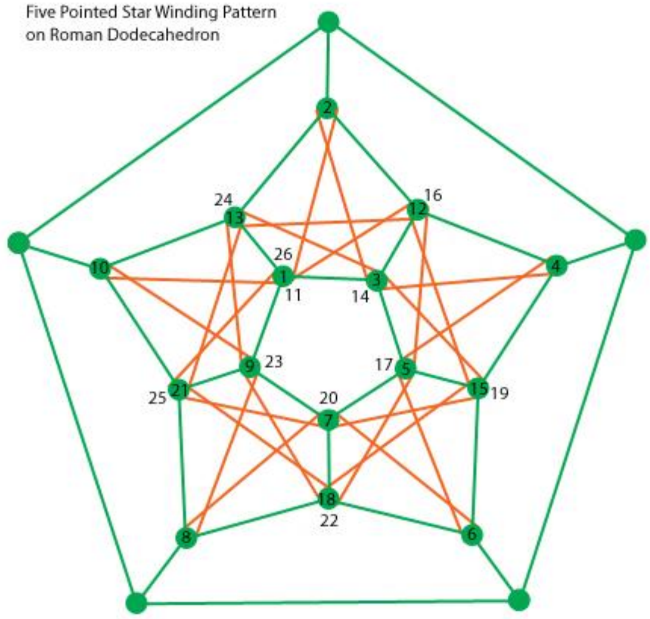


Figure 25 shows he circuit to create five five-pointed stars with the knobs numbered to show the order in which a thread can be wound around the knobs.

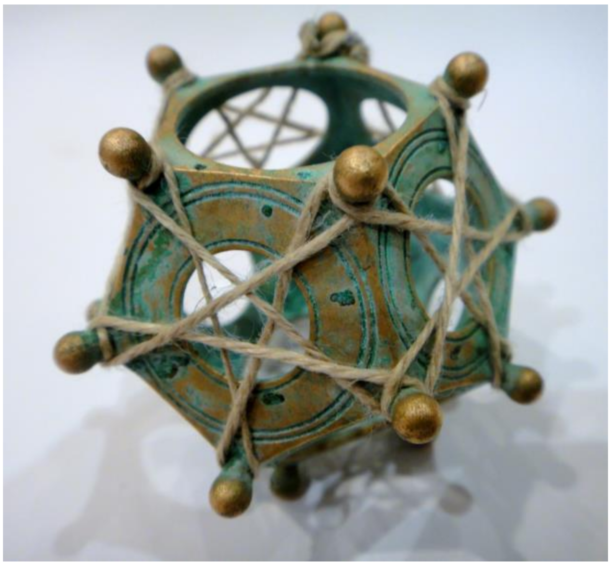


Figure 26 Replica 4cm Roman Dodecahedron wound with thread in the Five-star winding pattern. Length of twine required to complete the pathway was 88.8cm. Twine was wound each knob once to form each star and each knob is involved in the formation of two stars on adjacent faces.

The Jublains Dodecahedron with a pentagon edge length of 2.2 has a length between alternate knobs on each pentagonal face of 2 x 2.2 x sin54˚ = 3.56cm. Each pentagonal face therefore requires 17.8cm of thread to complete each pentagram. The five pentagrams that can be formed as a circuit require 17.8 x 5= 89cm of thread which is equivalent to approximately 2 Roman cubits (2 x 44.4cm) or 3 Roman Feet (Pedes) (3x29.6cm).

The pattern that creates five stars may have had some significance to the Romans. The Romans knew of five planets, namely in order of apparent luminosity as Venus, Jupiter, Mercury, Saturn and Mars, but they did not differentiate the planets from the stars except for the fact that they recognised that these planets which appeared like bright stars, from the reflected light of the Sun off their surfaces, appeared to move in a completely different fashion to the other stars and they were referred to as the five wandering stars. The other planets in the solar system are not visible to the naked eye. The word 'planet' comes from the ancient Greek ἀστήρ πλανήτης (astēr planētēs), meaning 'wandering star'.

The Roman Foot (the pes) and the Roman Cubit (the cubitus) appear to be the lengths obtained by winding twine around the dodecahedron knobs in the prescribed patterns.

Dodecahedron Distance between opposite pentagonal faces	Hamiltonian Circuit	Five Star Pattern
4cm	59.2cm (2 x 29.6cm)	88.8cm (2 x 44.4cm)
5cm	66.6cm (1.5 x 44.4cm)	118.4cm (4 x 29.6cm)

Table 27

Although the dodecahedrons have different sizes it can be seen how the Roman Foot and the Roman Cubit lengths can be obtained as multiple lengths from winding the different patterns.

Roman Dodecahedra as Rangefinders

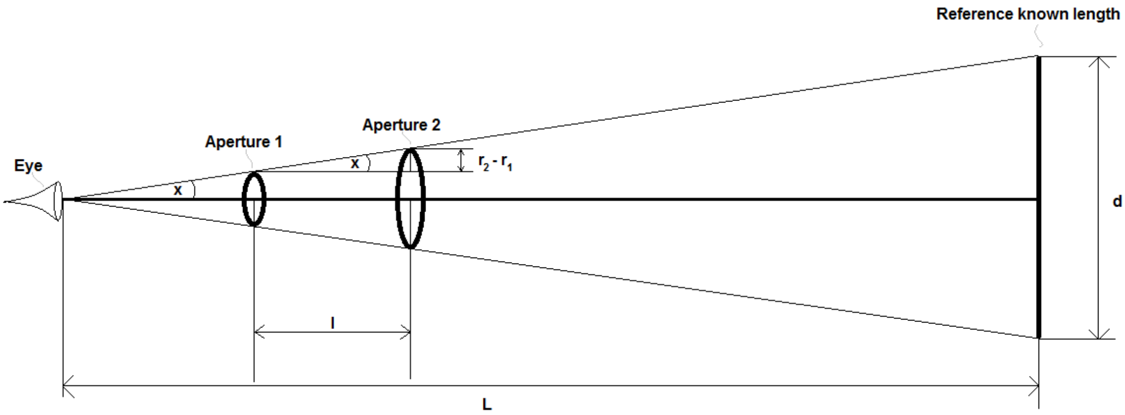
The Roman dodecahedrons have circular holes of varying diameters in their twelve faces. The holes in opposing faces have different sizes and it has been proposed that the series of circular apertures could be used as rangefinders or dioptra for surveying and the measurement of distance but the way in which this may have been achieved has not yet been described.

Several of the Roman Dodecahedra have been measured and their dimensions tabulated in *Roman Dodecahedron as dioptron: analysis of freely available data by Amelia Carolina Sparavigna at* https://arxiv.org/ftp/arxiv/papers/1206/1206.0946.pdf

Four examples of Roman Dodecahedra are listed below together with their dimensions. The artefacts were recovered from Jublains, Carnuntum, Tongre and Vienne. Each

Dodecahedron has a different size, the critical measurements are the diameters of the pairs of holes on opposing parallel faces and the distance separating those two faces. The size of the dodecahedra is expressed as the diameter of the circle on which the five vertices of the pentagon face, there is however no information regarding the twenty knobs on each corner of the dodecahedron so the winding of a twine cannot be determined without actually using the artefact itself. The angles viewed through the opposing holes can be calculated in order to determine how each dodecahedron might be used as a rangefinder.

By considering the difference between the radii (r_2-r_1) of opposing hole pairs and the distance separating these holes (l), the tangent of the angle "x" can be calculated ((r_2-r_1) /l). The angle x can be used to determine the distance to a vertically aligned known measurement by aligning the dodecahedron so that the two holes' circumferences appear to align with each other and the vertical measurement length (d). The distance between the eye and the vertical measurement length (L) can be calculated simply as L= d / 2((r_2-r_1) /l). The measurement length d used as a reference can be obtained by winding a twine around the knobs on the dodecahedra in the Hamiltonian circuit or Five Star winding pattern to obtain lengths of Roman Cubits or Roman Feet.



Method of Determining distance L between the eye and a known reference length d (measured in Roman Cubits or Roman Feet) using the Roman Dodecahedron as a dioptron or rangefinder.
The Dodecahedron is fixed and the eye is positioned so that the two aperture circumferences appear to coincide with each other. A known measurement length is then moved away from the viewer in a straight line until the ends of the length coincide with the line of sight of the aperture holes. The distance between the eye and the reference length can then be calculated as L= d / (2 x (r2 - r1) / l)

The ratio of (r_2 - r_1)/ l can be calculated for each Roman Dodecahedron by measuring the diameters of the aperture pairs and the distance separating them.

Diameter of Aperture (mm)	Diameter of Aperture (mm)	Difference in Radii of Apertures (mm)	Tan x $(r_2 - r_1) / l$	Distance L where d = 29.6cm	Distance L where d = 44.4cm
(25.5x21.5) (oval)	(26x21.5) (oval)	0.25	1/200	100 Roman feet	100 Roman Cubits
21.5	22	0.25	1/200	100 Roman Feet	100 Roman Cubits
16.5	17	0.25	1/200	100 Roman Feet	100 Roman Cubits
21	22	0.5	1/100	50 Roman Feet	50 Roman Cubits
11.5	15.5	2.0	1/25	12.5 Roman Feet	12.5 Roman Cubits
10.5	17.5	3.25	13/200	-	-

Table 28 Dimensions of Roman Dodecahedron from Jublains where l = 50mm

In other words in order to measure a distance of say 100 Roman Cubits, looking through the largest circular apertures, 21.5mm and 22mm, a pole with one Roman Cubit length marked on it could have been moved away from the viewing point until the length coincided with aperture, at which point the distance between the dodecahedron and the pole would be 100 Roman Cubits.

Diameter of Aperture (mm)	Diameter of Aperture (mm)	Difference in Radii of Apertures (mm)	Tan x $(r_2 - r_1) / l$	Distance L where d = 29.6cm	Distance L where d = 44.4cm
20.1	20.3	0.1	1/400	200 Roman feet	200 Roman Cubits
13.2	13.7	0.25	1/160	80 Roman Feet	80 Roman Cubits
21.4	22.4	0.5	1/80	40 Roman Feet	40 Roman Cubits
25	26.5	0.75	3/160	17 π Roman Feet	17 π Roman Cubits
15.3	17.3	1.0	1/40	20 Roman Feet	20 Roman Cubits
10.5	13	1.25	5/160	-	-

Table 29 Dimensions of Roman Dodecahedron from Carnuntum where l=40mm

Diameter of Aperture (mm)	Diameter of Aperture (mm)	Difference in Radii of Apertures (mm)	Tan x $(r_2 - r_1) / l$	Distance L where d = 29.6cm	Distance L where d = 44.4cm
16	16.2	0.1	1/630	Circumferenc e of circle of 100 Roman feet	Circumference of circle of 100 Roman Cubits
7.5	8.5	0.5	1/126	Circumferenc e of circle of 20 Roman Feet	Circumference of circle of 20 Roman Cubits
10.5	12.5	1.0	1/63	Circumferenc e of circle of 10 Roman Feet	Circumference of circle of 10 Roman Cubits
20	22.5	1.25	5/252	Circumferenc e of circle of 8 Roman Feet	Circumference of circle of 8 Roman Cubits
12.5	15.5	1.5	3/126	21 Roman Feet	21 Roman Cubits
12.5	16.5	2	2/63	Circumferenc e of circle of 5 Roman Feet	Circumference of circle of 5 Roman Cubits

Table 30 Dimensions of Roman Dodecahedron from Tongres where l=63mm

Diameter of Aperture (mm)	Diameter of Aperture (mm)	Difference in Radii of Apertures (mm)	Tan x $(r_2 - r_1) / l$	Distance L where d = 29.6cm 1 Roman Foot	Distance L where d = 44.4cm 1 Roman Cubit
23	24	0.5	1/88	Circumference of circle of 14 Roman Feet	Circumference of circle of 14 Roman Cubits
19	22	1.5	3/88	Circumference of circle of 42 Roman Feet	Circumference of circle of 42 Roman Cubits
14	20	3.0	3/44	Circumference of circle of 2.333 Roman Feet	Circumference of circle of 2.333 Roman Cubits
14	20	3.0	3/44	Circumference of circle of 2.333 Roman Feet	Circumference of circle of 2.333 Roman Cubits
15	22	3.5	7/88	Circumference of circle of 2 Roman Feet	Circumference of circle of 2 Roman Cubits
13.5	22	4.25	17/156		

Table 31 Dimensions of Roman Dodecahedron from Vienne where l=44mm

The dodecahedra from Jublains and Carnutum appear to be useful tools in measuring long multiple lengths of a reference length marked on a sighting pole. The two other dodecahedra from Tongres and Vienne appear to be different in that they appear to mark out circumference lengths of circles measured in Roman feet or cubits. The possible usefulness of being able to obtain a circumference length of a circle could be that a length of cord equivalent to the circumference length could be measured out and then easily divided into a desired number of parts that could then be laid onto the circumference of a marked circle on the ground of that circumference to mark out angles of arc or position equally spaced posts for construction purposes for instance.

Roman Foot and Roman Cubit used as Pendulums

Apart from their physical lengths that were used to measure distance on the ground, given what we have found about the ancient geodesic stone spheres, there is the possibility that when used as pendulum lengths that these Roman lengths could have periods that could be used to measure important time intervals as described by angles of the Earth's rotation measured as Megalithic Degrees (where 366 Megalithic Degrees = 360 degrees).

The period of a pendulum can be described by

$$T = 2\pi \sqrt{(l / g)}$$

where

T = period (s)

l = length of pendulum from point of suspension to the centre of the bob

g = gravitational field at latitude where used (m/s^2)

The lengths tested as pendulums were 29.6cm. 59.2cm, 44.4cm and 88.8cm,that is one Roman foot, 2 Roman feet, 1 Roman cubit and 2 Roman cubits.

For sidereal time, one Megalithic degree of rotation of the Earth relative to the stars takes 235.42 seconds. For solar time, one Megalithic degree of rotation of the Earth relative to the Sun takes 236.0656 seconds. The period of the different pendulum lengths can be calculated to determine whether a whole thousand number of swings coincides with an integer multiple of Megalithic degrees of rotation either relative to sidereal or solar time.

Pendulum Length l (m)	Period T (s)	Number of Swings / Angle (Meg Degrees) Solar Time	Number of Swings / Angle (Meg Degrees) Sidereal Time
0.296	1.0918	8000 / 37.00 MD	8000 / 37.10 MD
0.592	1.0091	2000 / 13.08 MD	2000 / 13.12 MD
0.444	1.3367	3000 / 16.99 MD	3000 /17.034 MD
0.888	1.8904	2000 / 16.02 MD	1000 / 8.030 MD

Table 32

It seems strange to consider that the Roman lengths have never been considered as possible pendulum lengths before but it does require that the concept of a circle being divided into 366 Megalithic Degrees as opposed to 360 “modern”degrees is understood for a whole number of swings to be achieved by these Roman pendulums for certain integer values of arc. Again, the property of being useful in measuring distance and time allows the lengths to be accurately calibrated, using the Sun in this case, without the requirement for comparison with a standard physical ruler.

The pendulums give the closest match to whole thousands of swings for solar time which makes sense if the dodecahedra were used as surveying instruments then they would have been used during the day unlike the Scottish pendulum lengths which were used both for sidereal and solar time but which were predominantly used for stellar measurements.

The value of angles which correspond to whole thousands of swings may appear like random values but 13,17 and 37 are hypotenuse values of three Pythagorean triplet right-angle triangles (5,12,13) (8,15,17), (12,35,37) and 16 is the length of two of the sides of the isosceles triangle (16,16,25). It is possible that right angle triangles whose three sides consisted of multiple integers were regarded as special triangles that could be used to measure right angles on the ground and establish a reference frame with respect to the cardinal positions of North, South, East and West when used in conjunction with the detection of the direction of the shortest shadow of a vertical stick when the Sun is due South in the Sky.

The Roman Cubit and the Roman Foot could have been accurately calibrated by using its known period when used as a pendulum. The required number of thousands of swings for the pendulums during the time it takes for the Earth to rotate by angles of 13, 16, 17 and 37 Megalithic Degrees could be utilised to adjust the lengths of the pendulums to obtain very accurate measurement lengths. It is interesting that the pendulums seem to be based on solar time and perhaps the pendulums were calibrated during the day and three poles for alignment with the Sun, one pole used to align with the other two poles separated by the required angle. In the cases of 13, 16,17 and 37 Megalithic degrees, these angles could be marked on the ground using an evenly spaced knotted cord to form triangles or arcs as described in the following tables.

Right angle triangles that could be used to obtain angles of 13, 16,17 and 37 megalithic Degrees.

Angle Required (Megalithic Degrees)	Length X	Length Y	Angle at vertex opposite Y (MD)	X+Y
13	22	5	13.02	27
16	32	9	15.97	41
17	10	3	16.98	13
37	19	14	36.99	33

Table 33

The usefulness of these right-angle triangles is limited by the fact that for these triangles the hypotenuse is not described by an integer multiple like the other two sides as with the Pythagorean triplet right angle triangles. For this reason a triangle such as the 3, 4, 5 triangle or the 5, 12, 13 triangle for example would need to be laid on the ground first to establish a reference for the right angle in order for the "X" and "Y" values to be properly laid out allowing the required angles to be accurately marked on the ground. There is one exception to this with the 16MD angle an alternative right angle triangle with a vertex angle of 32 Megalithic Degrees could have been laid out that was close to a Pythagorean triplet since an 8, 11, 21 gives a very close approximation (31.95MD) but would require 4000 swings of the 2 Roman Cubit pendulum (88.8cm) to be counted.

A triangle consisting of sides of known integer lengths is simpler to lay out on the ground and gives a better degree of accuracy in establishing the required angles for calibration of the pendulum lengths. Apart from the Pythagorean triplet triangles, Isosceles triangles consisting of multiple integer lengths for each side can be found that can be used to mark the required angles to allow the (29.6cm, 44.4cm, 59.2cm and 88.8cm) pendulums to be calibrated.

Angle (Megalithic Degrees)	Side Length (Two sides)	Base Side Length	Angle (MD)
13	9	2	12.97
16	11	3	15.94
17	55	16	17.01
37	33	19	37.01

Table 34

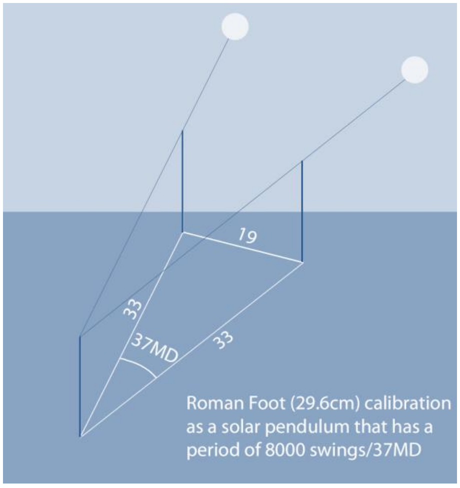


Figure 27

It can be seen that either method gives triangles that are suitable to use as reference angles to allow accurate calibration of the four pendulum lengths. The advantage of the isosceles triangles lies in the fact that all three sides of the triangle comprise of multiple integer lengths so that the angles can be marked in one operation without needing to establish a right angle.

There is a third way in which an equally spaced knotted cord could be used to mark the angles required to calibrate the four pendulums namely establishing an arc.

Angle (Megalithic degrees)	Radial Length	Arc length	Measured Angle (MD)
13	9	2	12.94
16	40	11	16.02
17	24	7	16.99
37	11	7	37.07

Table 35

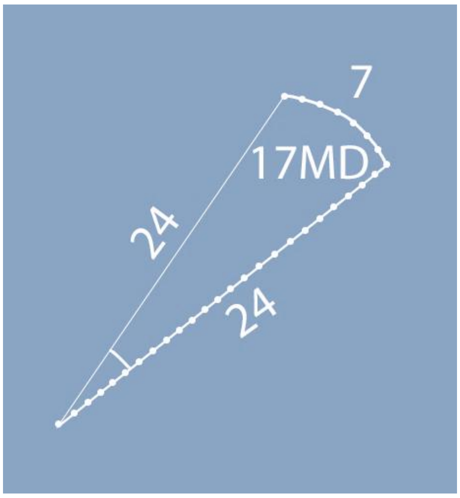


Figure 28

The way in which the triangles and arcs could be used in practice would have been to erect three straight vertical poles, one at each of the vertices marking the required angle. The pole at the tip of the required angle was the reference pole where the viewer would stand, North of the other two poles and this pole would be made to align firstly with the eastern-most pole to the South and the Sun and then the number of swings of the pendulum would be counted until the second western-most pole to the South aligned with the Sun from the same viewing pole as the Sun appeared to travel from East to West as the Earth spun about its axis. The pendulum length could be lengthened or shortened until the required number of swings was obtained but each time the position of the triangle or arc on the ground would need to be changed to allow the alignment of the Sun with the poles to be made. The four pendulum lengths could also be calibrated in another way at night time if star pairs could be identified that were separated by the precise angles required for calibration. The angles required are the same angles 13, 16,17 and 37 Megalithic Degrees but rather than solar time the movement of the Earth relative to the stars is known as sidereal time. The calibration could take account of this by choosing star pairs allowing 236.07 seconds per Megalithic degree of rotation based on sidereal time instead of the 235.42s required per Megalithic Degree of movement of the Sun. The following star pairs could have been for calibrating the four pendulums around 200AD in Northern France.

Star Pair	Constellations	Separation (Hour Angle) Required	Actual Separation (200AD)	Percentage
Aldebaran Pleiades	Taurus Taurus	13 MD 51m 9s	12.96MD 50m 59s	99.69
Alpheratz Markab	Andromeda Pegasus	16 MD 62m 57s	16.01MD 63m 0s	100.06
Sirius Betelgeuse	Canis Major Orion	17MD 66m 53s	17.01MD 66m 55s	100.06
Regulus Procyon	Leo Canis Minor	37MD 2h 25m 34s	36.99MD 2h 25m 32s	99.97

Table 36

It is interesting that the star pairs involve the brightest stars in their respective constellations Aldebaran is the brightest star in Taurus, Alpheratz the brightest star in Andromeda, Markab the brightest star in Pegasus, Sirius the brightest star in Canis Major, Betelgeuse the brightest star in Orion, Regulus the brightest star in Leo and Procyon the brightest star in Canis Minor and the Pleiades the brightest nebula in the night sky. The manner in which these star pairs could have been used to accurately calibrate the four Roman pendulum lengths was to erect two poles in the ground that aligned with the eastern-most star of the star pair and count the number of swings the pendulum took before the second star in the pair aligned with the same two poles. The length of the pendulum could be adjusted until it gave precisely the required number of swings. The possible accuracy of determining the Roman Foot and the Roman Cubit using these star pairs is within a fraction of a millimetre. Regulus-Procyon measures the Roman Foot, Sirius-Betelgeuse measures the Roman Cubit, Aldebaran- Pleiades measures 2 Roman Feet and Alpheratz-Markab measures 2 Roman Cubits. The bronze dodecahedron was ideally suited for use as the pendulum bob as a length of twine with a small loop tied at one end, big enough to fit over the small knobs , could be suspended by passing the end with a loop down through one of the holes and then back up and around one of the knobs around the hole through which the twine was passed through. The distance between the knob and the hole is approximately equivalent to the distance from the hole in the dodecahedron's side to the centre of gravity of the dodecahedron.

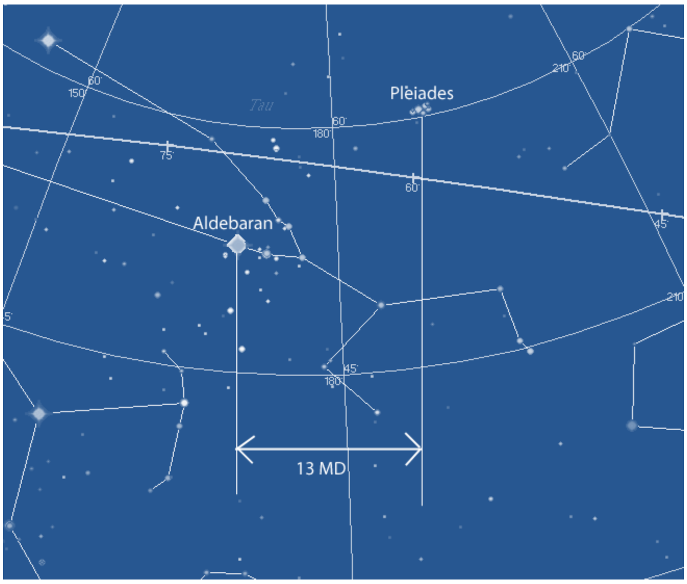


Figure 29

The Pleiades and Aldebaran in Taurus are separated by 50m 59s or 12.96 Megalithic Degrees. A pendulum of 59.2cm length of 2 Roman Feet will oscillate 2000 times during the time it takes for the alignment of the Pleiades to be replaced by the alignment of Aldebaran

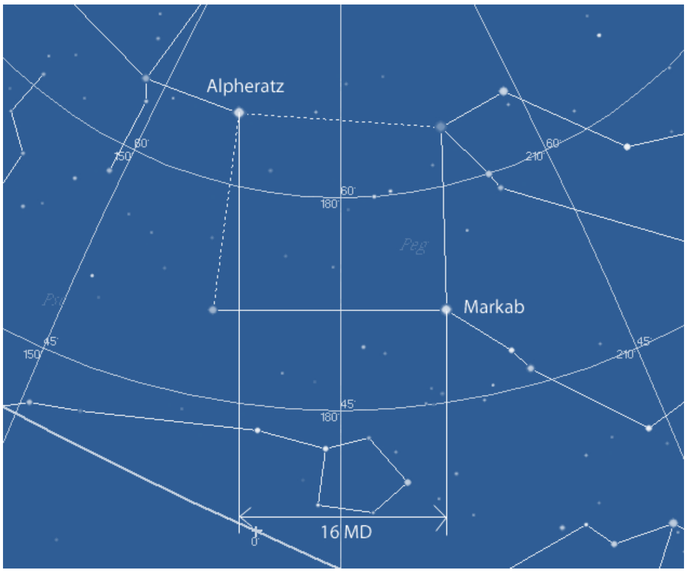


Figure 30

The stars Markab in Pegasus and Alpheratz in Andromeda are separated by 16 Megalithic Degrees that can be used to calibrate a pendulum of 88.8cm or 2 Roman Cubits of length which oscillates 2000 times for the time it takes for an alignment with Markab to be replaced with an alignment with Alpheratz

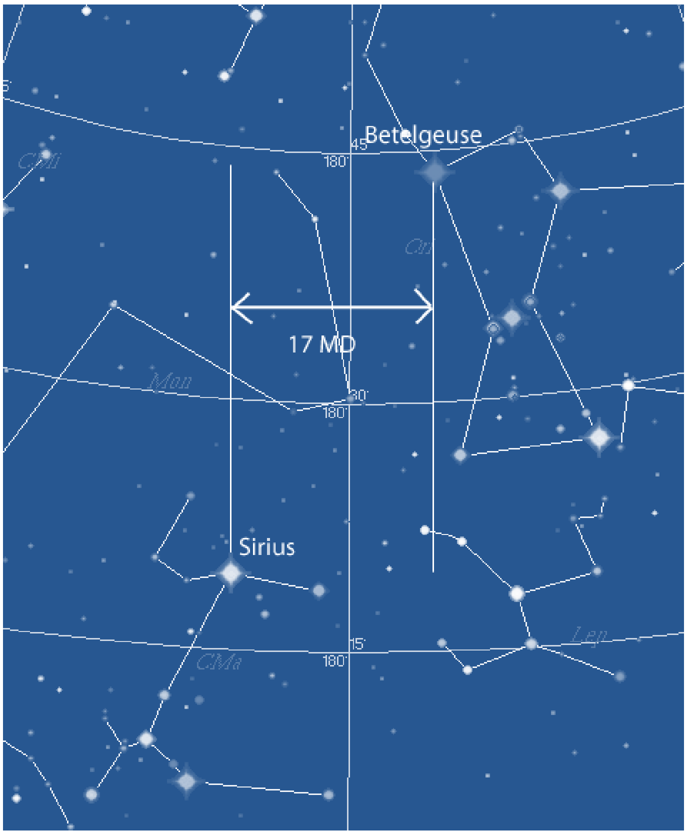


Figure 31

The stars Betelgeuse in Orion and Sirius in Canis Major are separated by 17 Megalithic Degrees that can be used to calibrate a pendulum of 44.4cm or one Roman Cubit which oscillates 3000 times for the time it takes for an alignment with Betelgeuse to be replaced by an alignment of Sirius

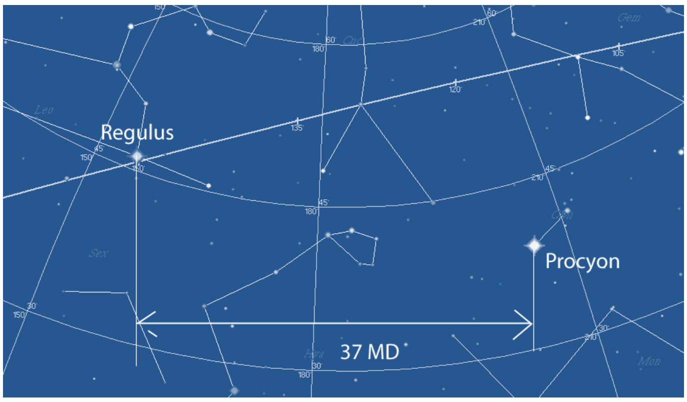


Figure 32

The stars Procyon in Canis Minor and Regulus in Leo are separated by 37 Megalithic Degrees that can be used to calibrate a pendulum of 29.6cm or one Roman Foot length which oscillates 8000 times for the time it takes for an alignment with Procyon to be replaced by an alignment with Regulus.

The Roman dodecahedron with its twenty small knobs can be used as a winding device to determine lengths of the Roman Foot and the Roman Cubit when twine is wound around the dodecahedron in the prescribed patterns. The holes of different sides can be used as rangefinders to determine longer distances when a vertical known length coincides with the apparent diameters of the two sighting holes. The third way in which these dodecahedra could be used was to measure time intervals when the Roman foot, 2 Roman feet, the Roman Cubit and 2 Roman Cubits were used as pendulums to measure time intervals expressed as angles of rotation of the Earth relative to the Sun in Megalithic degrees

Conclusions

It appears that around 3000BC people tried to make sense of their lives through observing what they could see in the sky each night of the year and relate that to their lives. The fact that the Earth rotates about its axis and revolves around the Sun means that apart from the Sun's varying seasonal path across the sky there is an ever-changing daily pattern of stars that appears to move across the sky but a pattern that has an annual cycle. Certain constellations have arrangements of stars that can be interpreted as animals and birds and fish or other objects that were important to the people of the time and the association of these star patterns with certain animals for instance are likely to have been influenced by the appearance of certain migrating animals at times of the year coinciding with the appearance of these constellations in the night sky. The passing of time was intimately connected with the seasons and the lives of the people as farmers and hunter gatherers and a method of measuring time and distance was developed to allow the people to form a calendar which helped people to organise their lives and give them a belief system where festivals dedicated to a pantheon of stellar deities could be celebrated. As part of this creation of a calendar, stone circles were constructed which allowed people to observe the alignment of the Sun on the horizon with megaliths on the special festival days. The clever development of a measuring system that combined the measurement of time and distance in the same system was at the heart of connecting the people with their star gods. The most important tool that was developed to allow measurement was the pendulum that consisted of a stone bob on a twine. The pendulum could be calibrated using its period of oscillation and adjusting its length until it gave the required number of swings for the time it took for the Earth to revolve by a known angle with reference to the stars or Sun. A simpler, more convenient way to obtain the various pendulum lengths was developed by fashioning stone spheres and stone geodesic spheres that allowed a string to be wound around various number of times to produce pendulum lengths in the form of multiple lengths that could be used to plan circular structures such as the stone circles and circular huts. A thousand years later wooden boards were made that acted as formers and allowed clay balls of the required size to be made without the need to spend hundreds of hours carefully chipping away at stones. A thousand years on, the Romans developed bronze dodecahedrons that allowed a string to be wound around knobs located at the vertices of the dodecahedron to form lengths of Roman pendulum lengths. Holes in the faces of the dodecahedra allowed the bronze dodecahedra to be used as dioptra.

In conclusion it is proposed that there have been a number of winding measurement devices over the past five millennia and that from 3000BC to 850AD that have been forgotten despite some of them surviving as stone and bronze artefacts. The first winding devices were made from stone and most of them were initially spherical but these were developed to form geodesic spheres carved with discs that allowed a twine to be wound around to give integer multiple lengths of all 12 pendulum lengths. Nearly two millennia later it appears that the same measurement system was being used but now a wooden former may have been made that allowed clay balls to be made easily which served a similar function to the stone balls. A thousand years later the Romans, rather than using a stone or clay ball, made bronze dodecahedrons with holes on opposite faces of the dodecahedron that allowed their pendulum lengths to be obtained by winding a twine around small knobs located at the vertices and which allowed the holes to be used as a sighting device for making measurements over longer distances. Finally, around 850-900AD in Scotland evidence carved on the Class II Pictish symbol stones suggests that a crank shape winding device may have been used to measure multiple lengths of Royal Cubits by winding a string around a series of quarter-arc, petal-shaped protuberances.

Appendix

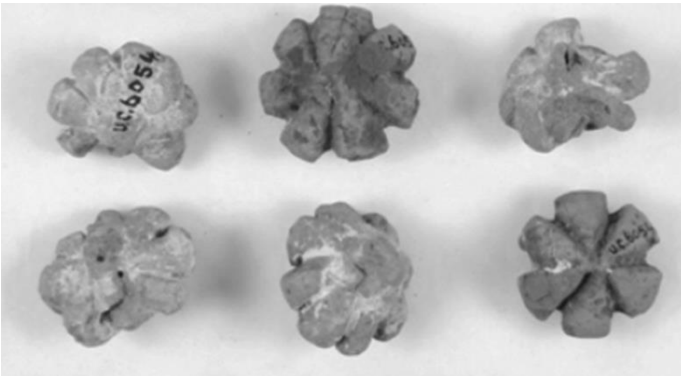
There is some evidence that the stone balls proposed as winding devices and rulers were used in other countries far removed from Scotland in pre-historic times.

Tiwanaku in Bolivia has a stone ball on display at its museum that shares a remarkable resemblance to the geodesic spheres found in Scotland. The ball has not yet been measured in the way proposed in this paper.

Another almost perfectly spherical stone ball found in North America has a circumference of 31.8cm, so that two windings around the ball carried out in the way that was done for the spherical balls in the National Museum of Scotland's collection, namely two perpendicular windings, results in a cord length of 63.6cm or what the Ancient Egyptians called a Sacred Cubit. This stone sphere was reportedly found in the mud of the Merrimack River in New England and advertised on e-Bay as a stone cannonball and was purchased by the author to see whether it could have been used as a winding device. This stone ball has been kept for safe-keeping until the significance of these stone balls is accepted by academia for what they are so that they can be properly displayed in museums and used to help understand the dimensions of archaeological structures constructed in pre-historic times and possible cultural links possibly resulting from sea travel between the continents.

Stone ball found in the Merrimack Riverbed in New England, U.S.A.

It is also interesting to see how the pendulum lengths used in Scotland from Neolithic times were adjusted for use in Egypt around 2500BC as evidenced by the close fit between the slightly shortened pendulum lengths (to account for the different gravitational field) with the dimensions of the pyramids of Giza. Although there are straight rulers that have survived from Ancient Egypt that appear to correspond with the Royal Cubit Measure, this was only one of many measurement lengths that was used at the time. It seems likely that the flexibility of the stone ball winding devices used in Scotland in being able to generate twelve different pendulum lengths by different winding regimes, would have been similarly useful in measuring multiple pendulum lengths in Egypt. The evidence for the existence of such winding devices comes in the form of unidentified artefacts from Ancient Egypt. Firstly there are stone objects that have been likened to garlic bulbs whose shape does share some similarity with the stone winding devices of Scotland.



Possible Stone winding devices from Ancient Egypt

Stone Ball and Bronze Double Hook retrieved from the North Shaft
of the Queens Chamber by Dixon in 1872.

Some Cedar wood recovered at the same time has recently been rediscovered in the Marischal University Museum in Aberdeen, Scotland. Carbon dating of the wood indicates an age of about 500 years older than the currently accepted date for construction of the Great Pyramid. The wood may have originally been attached to the bronze hook as a handle and have already been an ancient item at the time it was lost or deposited in the airshaft.

The stone ball and hook are now in the British Museum and the ball has a diameter reported as just under 4cm. This stone ball is smaller than the ones used in Neolithic Scotland but appears to be carved from granite. It should be noted that the ball does not appear to have the same level of sphericity as those from Scotland. The exact dimensions of the ball are not known but if the diameter is as reported as just under 4cm, the resulting circumference would be just under 12.56cm and it could be that four windings around the ball gave a pendulum length of 50.0cm. In which case the other pendulum lengths could be achieved